

Charged-lepton flavour violation and radiatively-generated neutrino mass

Innes Bigaran



1. Leptons in the Standard Model
2. Neutrino masses beyond the Standard Model
3. Radiative neutrino mass models
4. What about charged leptons?
5. A case study: a scalar leptoquark model for $g-2_s$

Leptons in the Standard Model

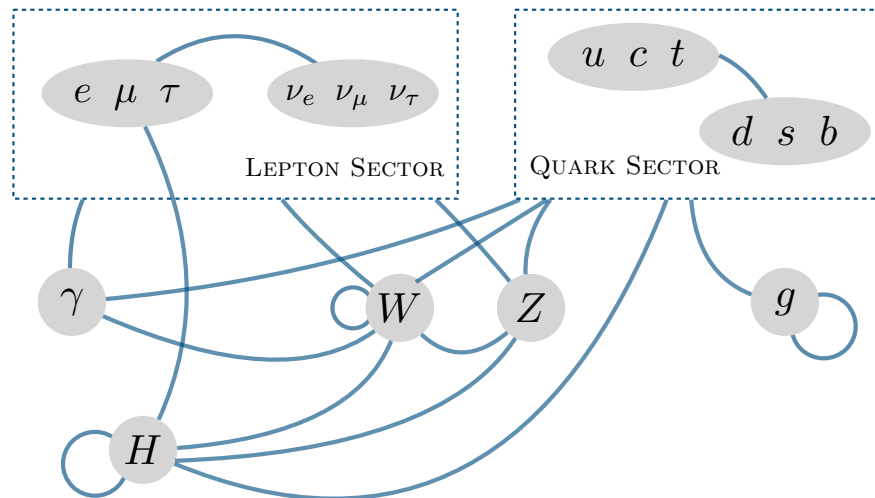
Standard Model interactions

	1st generation	2nd generation	3rd generation		
Matter	u	c	t	g	Mass
	d	s	b	γ	
	e	μ	τ	Z	Forces
	ν_e	ν_μ	ν_τ	W	
				H	

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

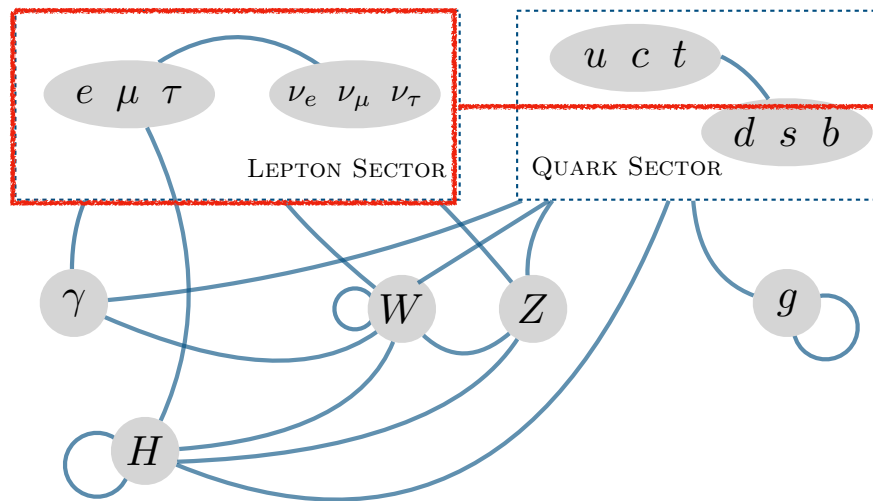
- SM matter sector: quarks and leptons
- Exist in three generations, where these generations differ only by their masses
- Successive generations are more massive
- In the SM, all neutrinos are massless.

Standard Model interactions



- Massive particles get their masses via a Yukawa interaction with the Higgs, which after EWSB gives them a **Dirac mass**
- SM neutrinos don't have this interaction

Standard Model interactions: lepton masses



- Lepton fields in the SM, pre EWSB

$$L_L \sim \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, -1/2)$$

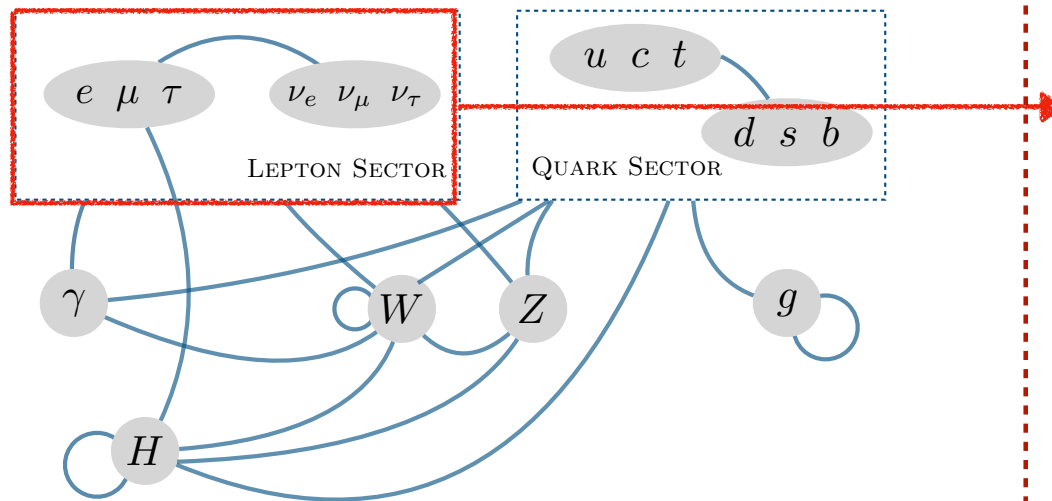
$$e_R \sim (1, 1, -1)$$

- One lepton mass term in SM:

$$\mathcal{L} \supset -y_e \bar{L}_L H e_R \xrightarrow[\text{gets a VEV}]{\text{EWSB, } H_0} -M_e \bar{e}_L e_R$$

- In the SM there's no RH neutrino to generate a **Dirac mass term** for the neutral leptons

Standard Model interactions: lepton symmetries



- If all the SM leptons were massless, the accidental global symmetry

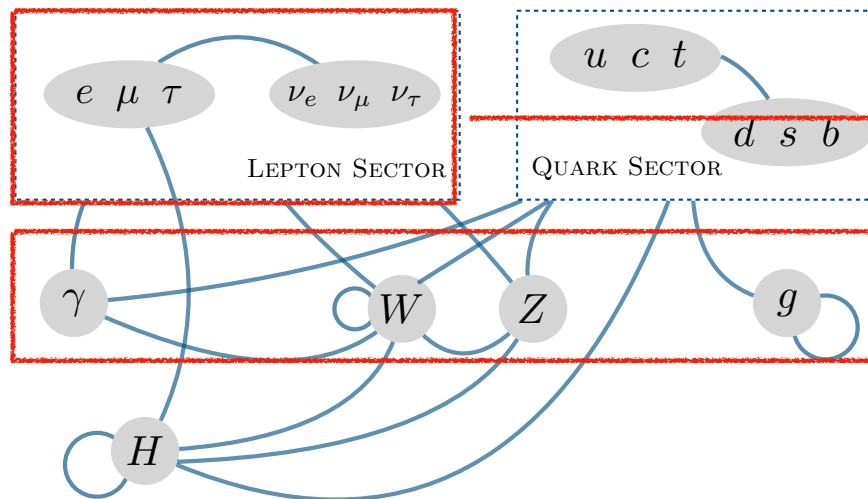
$$U(3)_{E_L} \otimes U(3)_{E_R}$$

- Lepton masses break this symmetry into the subgroup

$$\mathcal{G}_L = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

- Correspond to a perturbative conservation of **lepton flavour**. Total **lepton number (L)** (sum of flavoured QN) also conserved.

Standard Model interactions: lepton symmetries



- Move to a basis where the masses generated by EWSB are diagonal \rightarrow propagating fields correspond to the physical fermions
- Influence of this basis change on the gauge interactions? unitary rotations yield diagonal and *flavour-independent* couplings of the leptons to the gauge bosons
- This is “lepton flavour universality”

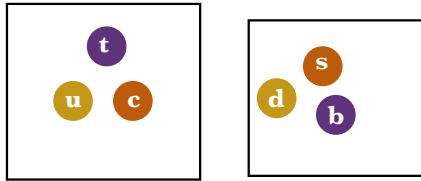
**NB: these rotations not ‘dropping out’ for the quarks is where the CKM comes in in the quark sector.

Neutrino masses beyond the SM

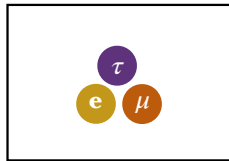
Fermion masses in the SM

- SM fermion masses are Dirac masses: corresponding to self-interactions between the RH and LH Weyl fermions, e.g. $\mathcal{L} \supset -M_e^{ij} \bar{e}_L^i e_R^j$
- Size of the mass is controlled by the size of the Yukawa interaction with H

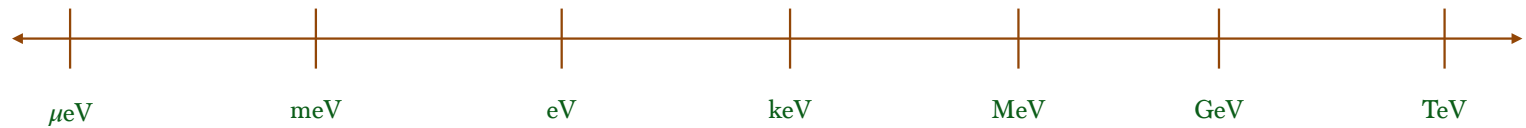
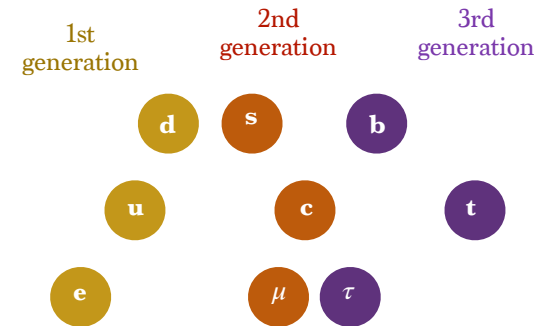
Up-type quarks



Down-type quarks



Charged leptons

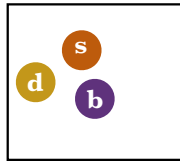
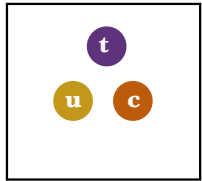


$$E = mc^2$$

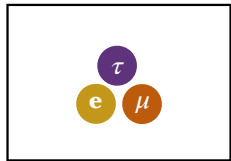
Fermion masses in the SM

- Surprise! we measure (small but nonzero) neutrino masses: **need new physics**
- We measure it via neutrino flavour oscillations: **new physics violates lepton flavour**

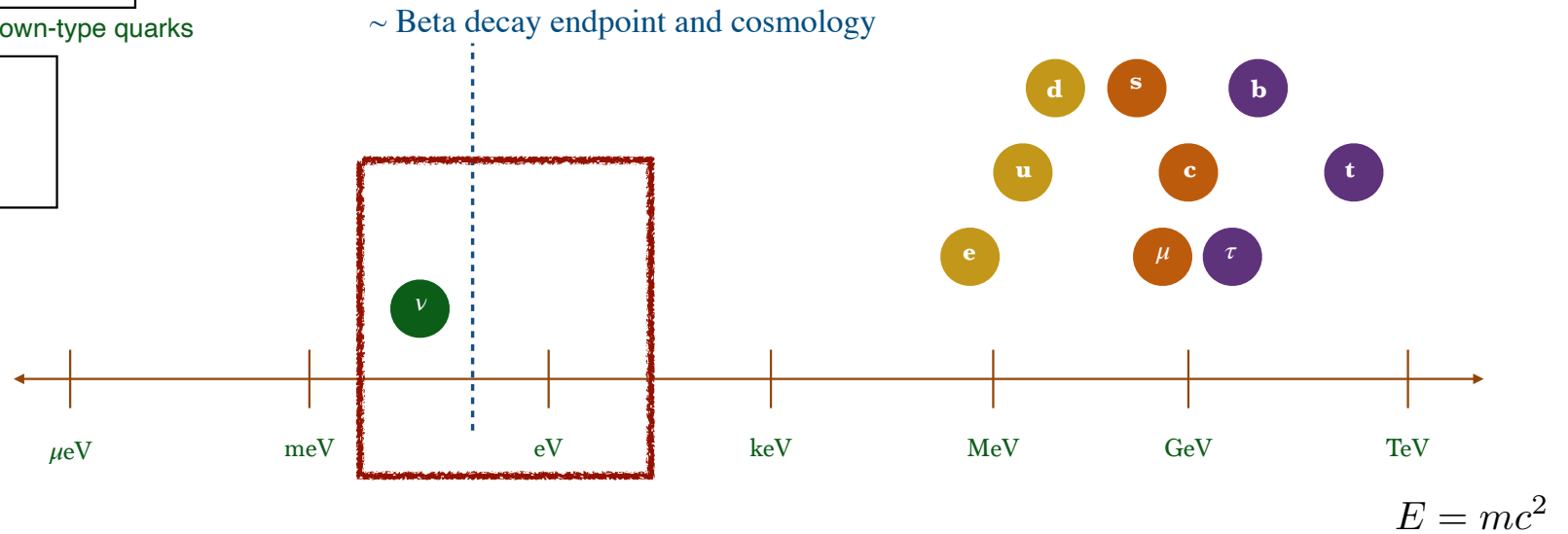
Up-type quarks



Down-type quarks



Charged leptons



ν mass = ν physics

- Neutrino masses could be either **Dirac** or **Majorana** in nature
- Dirac mass terms involve adding right-handed neutrinos (new d.o.f.) $\nu_R \sim (1, 1, 0)$
- Majorana masses involve a self-interaction between the neutrino and its charged conjugate:

$$\mathcal{L} \supset -M_{\text{maj}} \overline{\nu_L^C} \nu_L$$

- Majorana masses violates lepton number by two units: $\Delta L = 2$
- If neutrinos have Majorana masses, they are the first (of many?) Majorana particles to be discovered!

ν mass = ν physics: Majorana neutrinos

- For the remainder of this talk: let's **assume neutrinos are Majorana**, and we don't add RH neutrinos. If we are trying to build a model for neutrino masses we want to know:
 1. Why/how is L violated?
 2. Why are neutrino masses so small?

Origin of L violation

- Extending beyond the SM, write down effective interactions higher mass dimension (non-renormalisable)
 - These effective operators can be “opened-up” by introducing new field content, and writing down a complete model that generates that interaction
-

The Weinberg operator (D=5)

$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H \quad \longrightarrow \quad m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

λ is a dimensionless coupling
 Λ is a new “mass scale”
 v is the Higgs vev

Origin of L violation

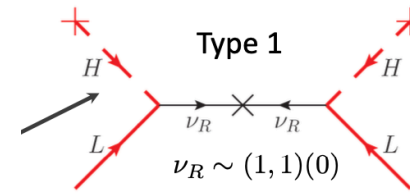
The Weinberg operator (D=5)

$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H$$

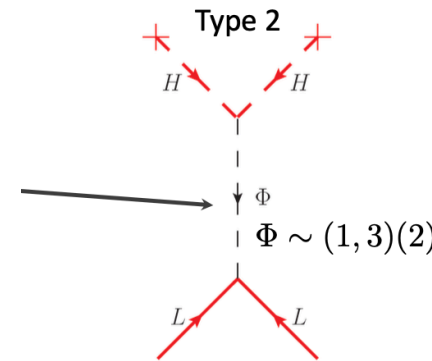
$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

If we 'open up' this operator at tree-level, we get the *Seesaw models*

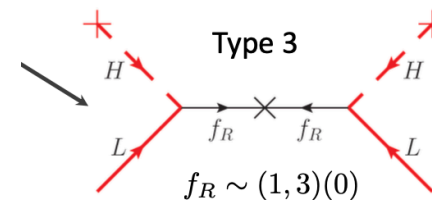
Here, your new fields have L violating interactions



Minkowski 1977
Yanagida 1979
Gell-Mann, Ramond, Slansky 1979
Mohapatra, Senjanovic 1980



Magg, Wetterich 1980
Schechter, Valle 1980
Cheng, Li 1980
Lazarides, Shafi, Wetterich 1981
Wetterich 1981
Mohapatra, Senjanovic 1981



Foot, Lew, He, Joshi 1989

R. Volkas, DSU 2022

Why are neutrino masses so small?

The Weinberg operator (D=5)

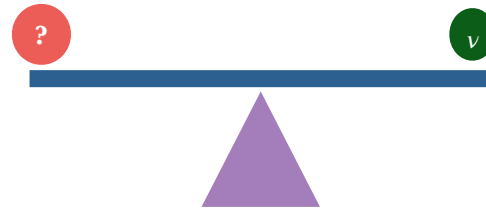
$$\mathcal{L}_{\text{effective}} \supset \frac{\lambda}{\Lambda} L_L L_L H H$$

↓

$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

- Observable neutrino mass scales are very small

$$\Lambda \gg v^2$$



Why are neutrino masses so small?

The Weinberg operator (D=5)

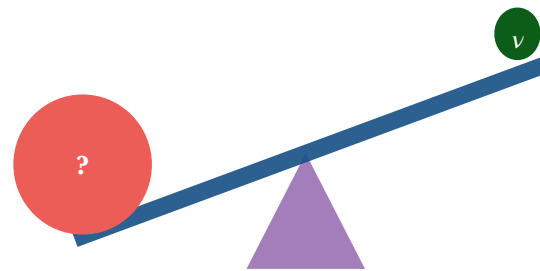
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$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

- Observable neutrino mass scales are very small

$$\Lambda \gg v^2$$

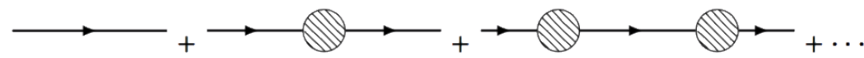
e.g. $m_\nu \sim 0.1 \text{ eV}, v \sim 10^2 \text{ GeV} \implies \Lambda \sim 10^{14} \text{ GeV}$



This makes these tree-level seesaw models generally hard to probe experimentally

There are variations on this which do allow more concrete testability (e.g. inverse and linear seesaw models)

Why are neutrino masses so small?



$$\begin{aligned}
 iG(\not{p}) &= \text{---} + \text{---} \circ \text{---} + \text{---} \circ \text{---} \circ \text{---} + \dots \\
 &= \frac{i}{\not{p}} + \frac{i}{\not{p}} [i\Sigma(\not{p})] \frac{i}{\not{p}} + \frac{i}{\not{p}} [i\Sigma(\not{p})] \frac{i}{\not{p}} [i\Sigma(\not{p})] \frac{i}{\not{p}} + \dots \\
 &= \frac{i}{\not{p}} \left(1 + \frac{-\Sigma[\not{p}]}{\not{p}} + \left[\frac{-\Sigma[\not{p}]}{\not{p}} \right]^2 + \dots \right) \\
 &= \frac{i}{\not{p}} \frac{1}{1 + \frac{\Sigma[\not{p}]}{\not{p}}} \\
 &= \frac{i}{\not{p} + \Sigma(\not{p})}.
 \end{aligned}$$

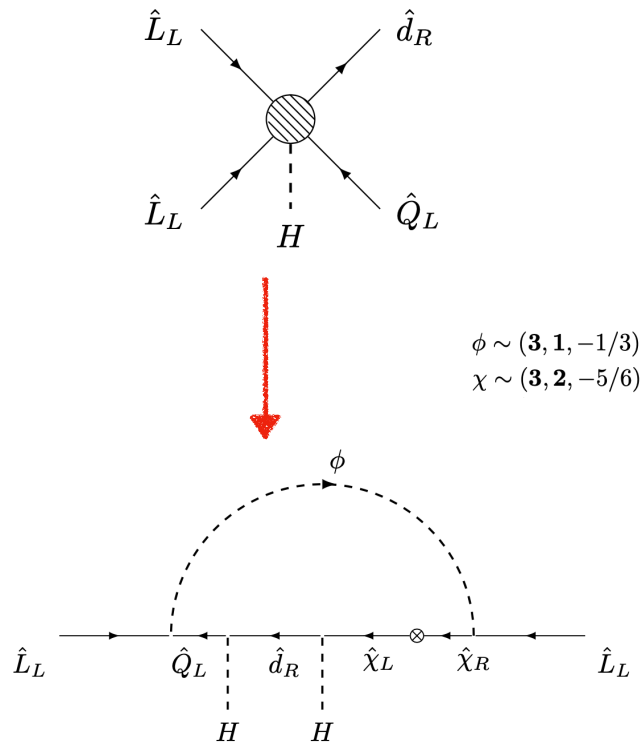
$$m_\nu = -\Sigma(p=0)$$

Correction to the neutrino self-energy

- If we say tree-level neutrino mass requires too high of an energy scale, what if we move to loop-level?
- The neutrino mass is no longer just suppressed by the mass scale, but also of the higher order of the interaction
- Can naturally lead to smaller neutrino masses!

Radiative neutrino mass models

Generating radiative neutrino mass



- One can systematically study $\Delta L = 2$ effective operators from the SMEFT
- Opening up (a.k.a. UV completing) these operators motivates particular SM extensions

- Generalised Weinberg operators, e.g.

$$\frac{\lambda}{\Lambda^{1+2n}} LLHH (H^\dagger H)^n$$

See, e.g. Hirsch, Cepedello et al (many papers)

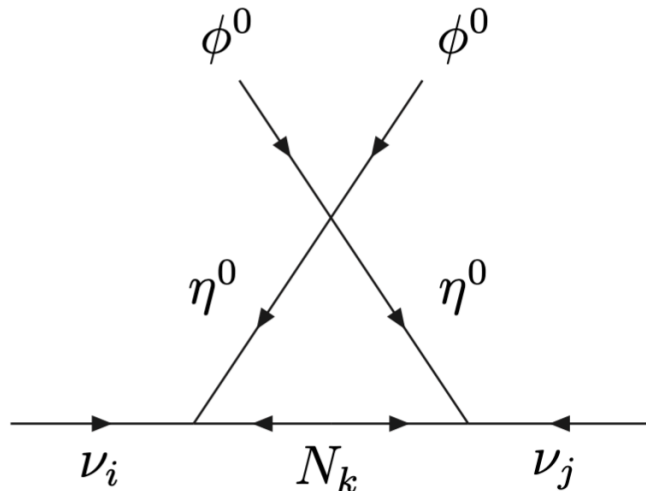
- non-Weinberg operators, e.g.

$$\frac{\lambda}{\Lambda^6} L_L Q_L L_L \bar{d}_R H$$

See, e.g. Babu and Leung '01, de Gouvea and Jenkins '08

Aside for those of you here for DM :

Generating radiative neutrino mass... with DM?



Ma 2006 arXiv:hep-ph/0601225

- *For example, the model aside:*
 - Heavy Majorana neutrinos, N
 - Scalar inert doublet, η
 - Both are Z_2 odd DM candidates
- *Scotogenic models* (scoto = dark)
- This is a type of “neutrinophilic” dark matter

Generating radiative neutrino mass

- Generalised Weinberg operators, e.g.

$$\frac{\lambda}{\Lambda^{1+2n}} LLHH(H^\dagger H)^n$$

- non-Weinberg operators, e.g.

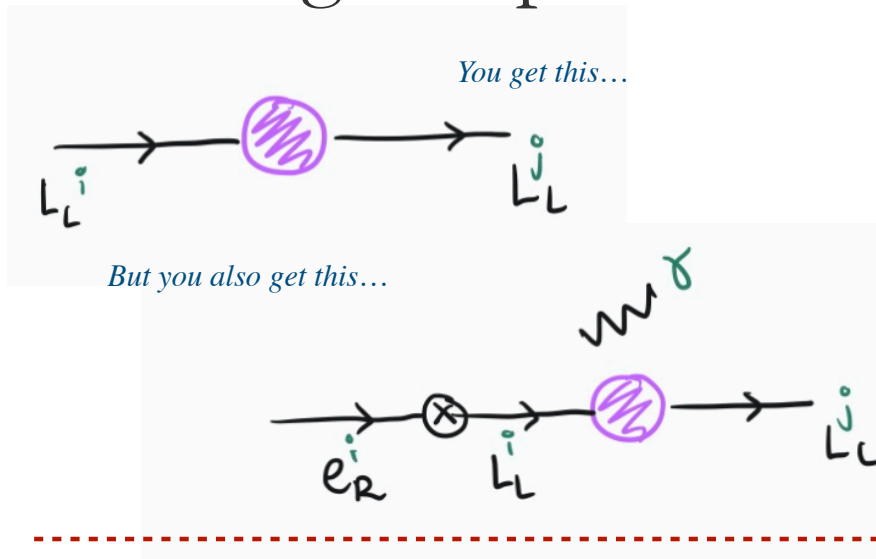
$$\frac{\lambda}{\Lambda^6} L_L Q_L L_L \bar{d}_R H$$

- When we look at these effective operators, note that we are talking about the SMEFT, at high energy
- Recall that the left-handed SM lepton doublet includes both the neutrino and charged lepton

$$L_L \sim \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, -1/2)$$

- Models that give neutrino flavour mixing will result in charged **lepton flavour violation** (cLFV).
- Not necessarily generated by the same effective operator in a UV-complete model

Charged-lepton flavour violation (cLFV)



- SM without neutrino masses conserves lepton flavour
- Neutrino masses consistent with neutrino flavour oscillation experiments necessarily lead to LFV
- Once this lepton flavour symmetry is abandoned, why not also see it in interactions of charged leptons?

Present and [Future] bounds on some key cLFV processes

$$\text{BR}(\mu \rightarrow 3e) < 1.0 \times 10^{-12} [\sim \times 10^{-16}]$$

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13} [6 \times 10^{-14}]$$

$$\text{BR}(\tau \rightarrow \mu\gamma) < 4.2 \times 10^{-8} [6.9 \times 10^{-9}]$$

$$\text{BR}(\tau \rightarrow 3\mu) < 2.1 \times 10^{-8} [3.6 \times 10^{-10}]$$

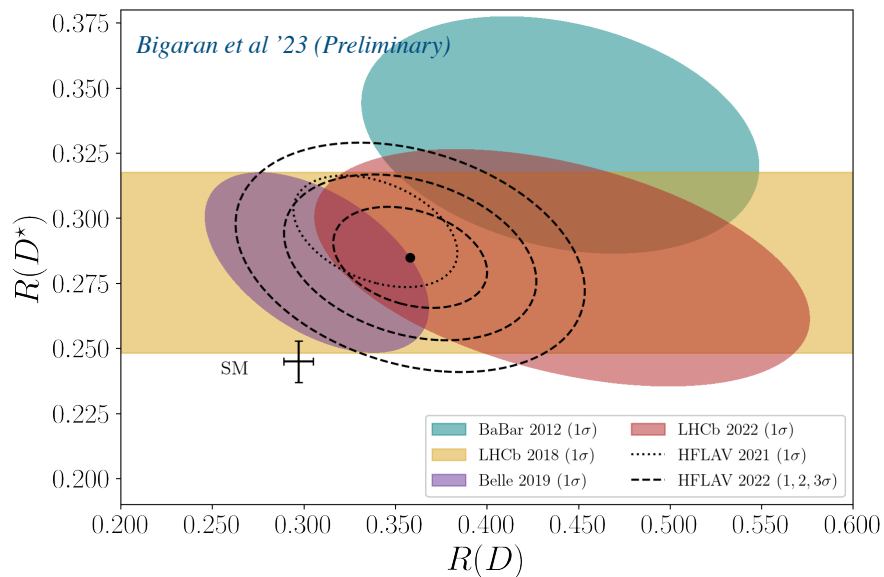
cLFV decays provide some of the strongest constraints on radiative neutrino mass models

What about charged leptons?

Lepton flavour universality and LFV

E.g. hints of LFU violation in B-meson decays:

$$R_{D^{(*)}} = \frac{Br(B \rightarrow D^{(*)} \tau \nu_\tau)}{Br(B \rightarrow D^{(*)} \ell \nu_\ell)}$$

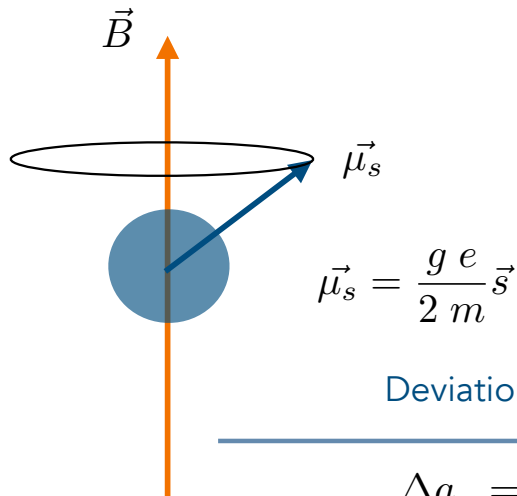


- Connecting cLFV and neutrino masses is not new, of course... *e.g. see Casas and Ibarra '01, Davidson et al. (many papers)*
- But we have many good ideas for models that predict LFV, which generates apparent LFU violation

‘Flavour anomalies’ in recent years have attracted a lot of attention, and many explanations have relied on lepton-flavour-specific couplings of new physics

Lepton flavour universality and LFV

E.g. the anomalous magnetic moments of the muon and the electron



Deviation from SM prediction	Significance
$\Delta a_\mu = (2.86 \pm 0.76) \times 10^{-9}$	4.2σ
$\Delta a_e^{\text{Cs}} = -(0.88 \pm 0.36) \times 10^{-12}$ <small>Parker et al 1812.04130</small>	2.5σ
$\Delta a_e^{\text{Rb}} = (4.8 \pm 3.0) \times 10^{-13}$ <small>Morel et al 2020, INSPIRE: 1837309</small>	1.6σ

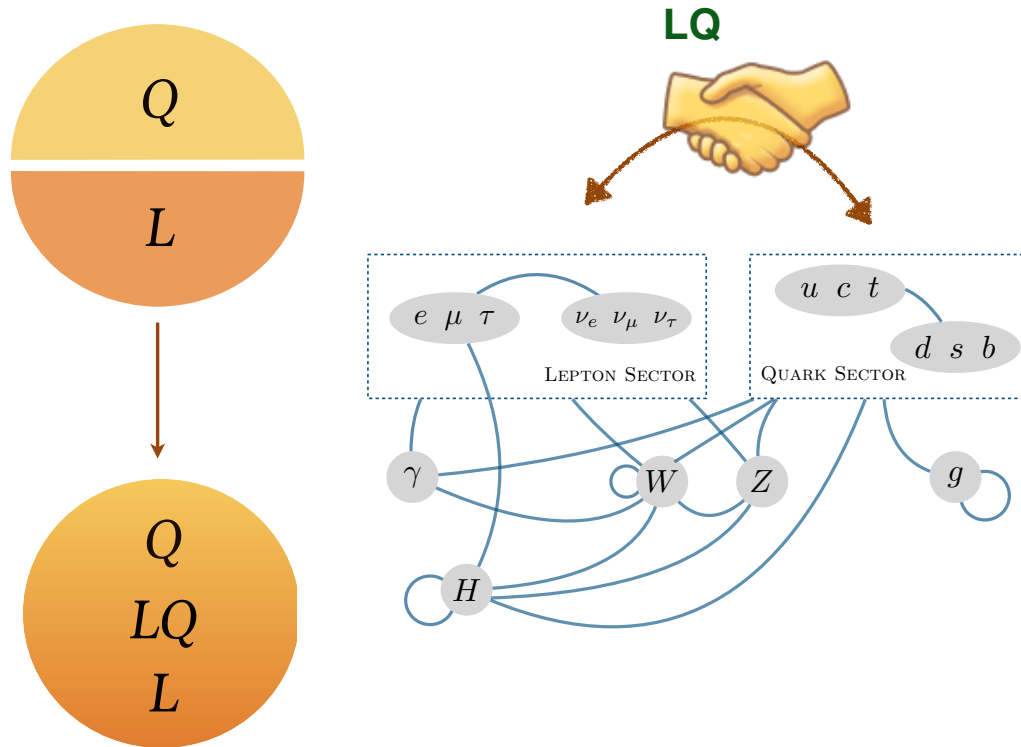
N.B. here I have assumed the R-ratio method for calculating SM prediction for HVP contribution to muon g-2

- Many of these flavour anomalies have faded to statistical insignificance over the years.... :(
- But the work done on trying to explain them doesn't become obsolete — we have very good motivation for flavour-specific new physics couplings in the lepton sector!

A case study: a scalar leptoquark model for $(g-2)$'s

Based on Bigaran, Volkas [2002.12544](#) and [2110.03707](#)

Scalar leptoquarks as a BSM candidate



Constraints from direct LHC searches imply at least **TeV scale masses**

- **Couple SM quarks and leptons directly**

e.g. charged lepton interactions

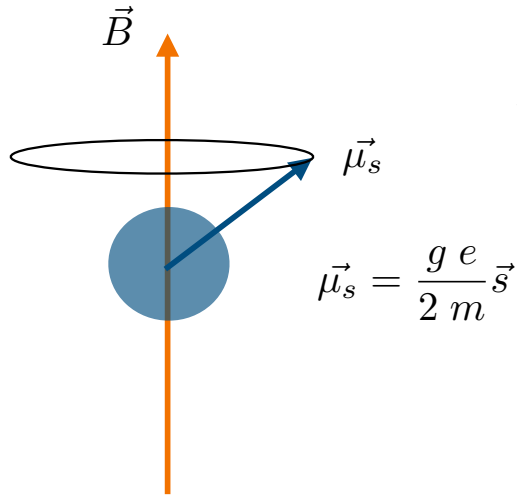
Leptoquark, ϕ

$$\mathcal{L}_\ell = \overline{\ell^{(c)}} [y^R P_R + y^L P_L] q \phi^\dagger + h.c.$$

Chirality of coupling labelled by chirality of q it couples to

- **Finite set of scalar leptoquarks**

Symbol	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
\tilde{S}_1	$(\mathbf{3}, \mathbf{1}, -4/3)$
S_1	$(\mathbf{3}, \mathbf{1}, -1/3)$
S_3	$(\mathbf{3}, \mathbf{3}, -1/3)$
\bar{S}_1	$(\mathbf{3}, \mathbf{1}, 2/3)$
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$
\tilde{R}_2	$(\mathbf{3}, \mathbf{2}, 1/6)$



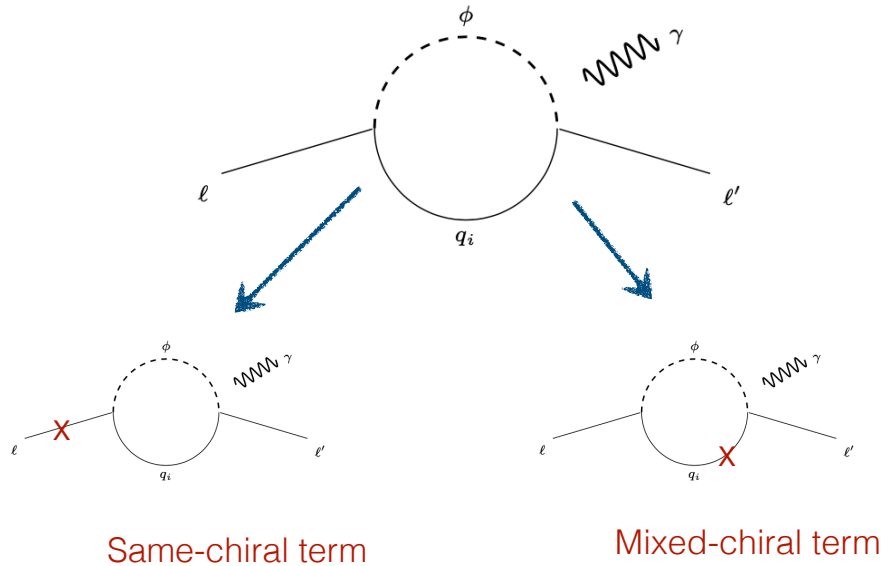
The problem

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N.B. here I have assumed the R-ratio method for calculating SM prediction for HVP contribution to muon g-2

- There are deviations from the SM in the anomalous magnetic moment of the electron and muon
- The electron presently has two measurements: one has a pull away from SM, one has a pull towards
- Electron and muon *may* require BSM corrections in opposite directions

A solution: scalar leptoquarks



Symbol	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
\tilde{S}_1	$(\mathbf{3}, \mathbf{1}, -4/3)$
S_1	$(\mathbf{3}, \mathbf{1}, -1/3)$
S_3	$(\mathbf{3}, \mathbf{3}, -1/3)$
\bar{S}_1	$(\mathbf{3}, \mathbf{1}, 2/3)$
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$
\tilde{R}_2	$(\mathbf{3}, \mathbf{2}, 1/6)$

Only mixed-chiral scalar LQ generate a mass-enhanced correction

e.g. charged lepton interactions

$$\mathcal{L}_\ell = \overline{\ell^{(c)}} [y^R P_R + y^L P_L] q \phi^\dagger + h.c.$$

Leptoquark, ϕ

- Contributions to the g-2 of a lepton $a_\ell = \frac{1}{2}(g - 2)$

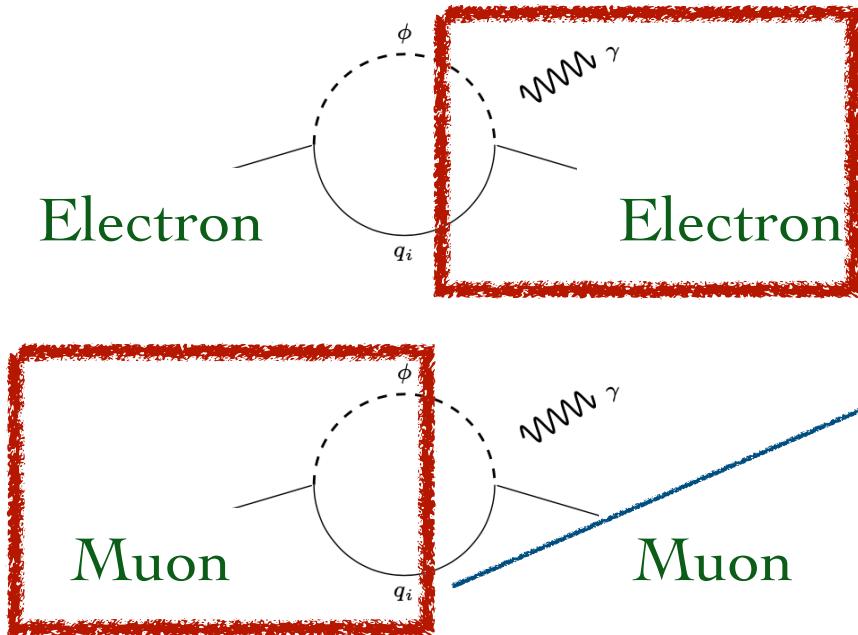
$$\Delta a_\ell = -\frac{3m_\ell}{8\pi^2 m_\phi^2} \sum_q \left[m_\ell (|y_\ell^R|^2 + |y_\ell^L|^2) \kappa + m_q \text{Re}(y_\ell^{L*} y_\ell^R) \kappa' \right]$$

κ and κ' are loop functions, dependent LQ and quark mass ratio

- If we need to generate opposite sign and enhanced pull from the SM value for different leptons, we need couplings of *both chiralities*

Scalar leptoquarks for the muon and electron g-2

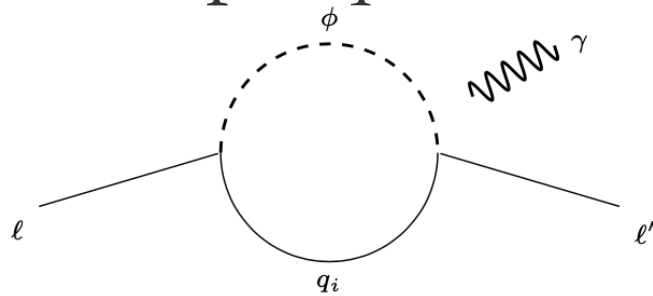
- But if we have the same intermediate quark, this mixed-chiral enhancement for the individual muon and electron dipole moments corresponds to an enhanced contribution to cLFV decays



$$\mu \rightarrow e\gamma$$

Need to couple muon and electron to different quark flavours to avoid this constraint!

Scalar leptoquarks for the muon and electron g-2



E.g. the S_1 scalar leptoquark

$$\sim (\mathbf{3}, \mathbf{1}, -1/3)$$

$$\mathcal{L}_{\text{int}}^{S_1} = (\overline{L}_L^c \lambda_{LQ} Q_L + \overline{e}_R^c \lambda_{eu} u_R) S_1^\dagger + h.c.,$$

Mixed chiral couplings to up-type quarks

$$\mathcal{L}_\ell = \overline{\ell^{(c)}} [y^R P_R + y^L P_L] q \phi^\dagger + h.c.$$

$$\mathbf{y}^L \sim \begin{pmatrix} 0 & \blacksquare & 0 \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{y}^R \sim \begin{pmatrix} 0 & \blacksquare & 0 \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{pmatrix}$$

$u \quad c \quad t$

$e \quad \mu \quad \tau$

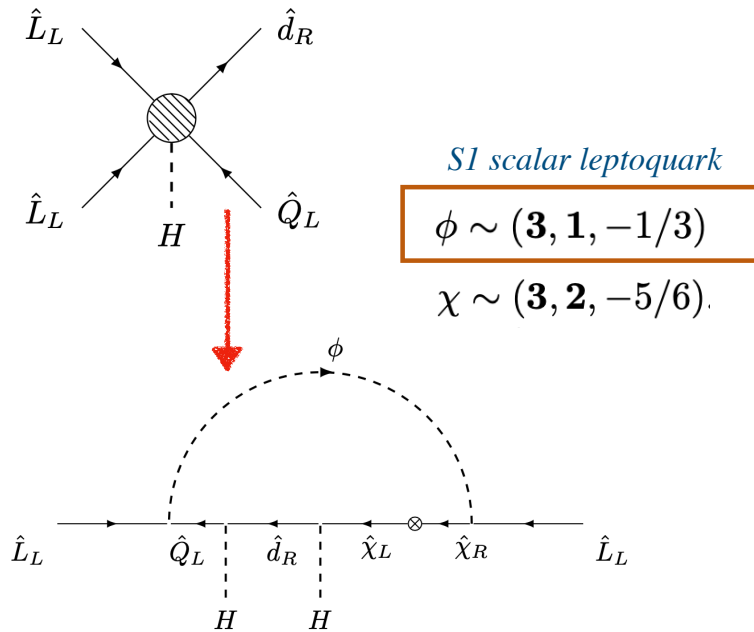
Row-column = lepto-quark, mnemonic

- Contribution to (g-2) of the electron is via a **charm**-containing loop
- Contribution to (g-2) of the muon is via a **top**-containing loop
- Constraints from cLFV in particular avoided by having different intermediate SM quarks coupling for each dipole vertex

$$\Delta a_\ell^{S_1} \sim -\frac{m_\ell m_q}{4\pi^2 m_{S_1}^2} \left[\frac{7}{4} - 2 \log \left(\frac{m_{S_1}}{m_q} \right) \right] \text{Re}(y_{\ell q}^{L*} y_{\ell q}^R),$$

“Okay, so why did you tell me this?”

Cai et al '17, Bigaran et al '19, Bigaran et al '23 (TBD)



- Mixed-chiral scalar leptoquarks can easily generate large contributions to cLFV processes at one-loop
- To avoid them, we need to play some ‘flavour games’ with their couplings (decoupling via intermediate quark couplings, up-type quark couplings in particular)
- These scalar LQ appear in other BSM models, including those for radiative neutrino masses (see aside)

$$\mathcal{L}_{\text{int}}^{S_1} = (\overline{L}_L^c \lambda_{LQ} Q_L + \overline{e}_R^c \lambda_{eu} u_R) S_1^\dagger + h.c.,$$

“Okay, so why did you tell me this?”

- Other flavour anomaly models particularly found tau decays to be very strongly constraining (e.g. S1 for b to c tau nu models that also address the muon g-2)
- We have learned a lot about how to avoid large contributions to cLFV in flavour models by building models that explain anomalies *See e.g. Bigaran, Hagedorn et al 2022*
- Also...

“Okay, so why did you tell me this?”

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- Also...

Learn a lesson from those who ambulance chase

What was once a constraint may one day become a target!



High-precision measurement of the W boson mass with the CDF II detector

CDF Collaboration · T. Aaltonen (Helsinki U. and Helsinki Inst. of Phys.) [Show All\(398\)](#)

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7 pages

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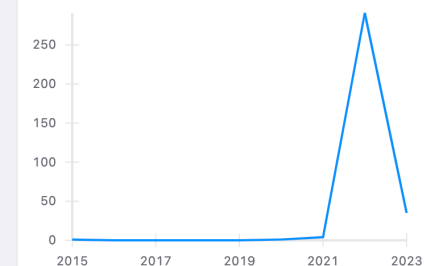
DOI: [10.1126/science.abk1781](https://doi.org/10.1126/science.abk1781)

Report number: FERMILAB-PUB-22-254-PPD

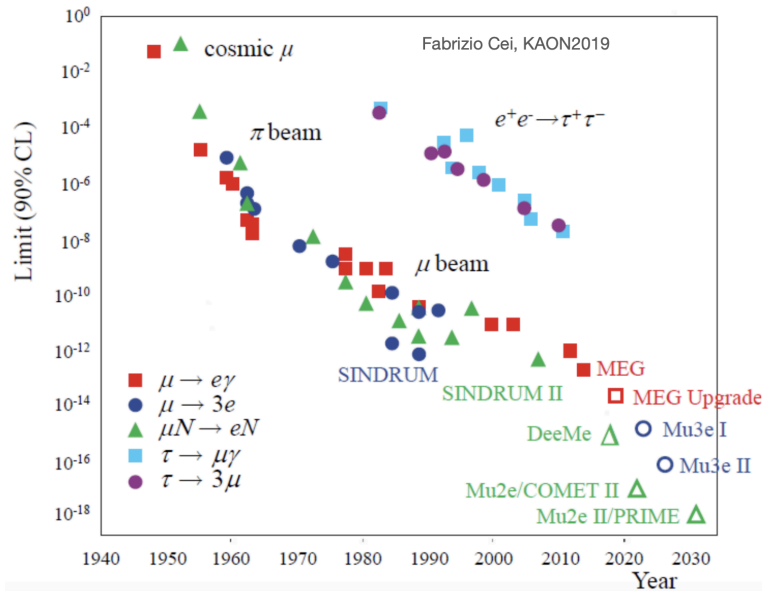
Experiments: [FNAL-E-0830](#)

View in: [OSTI Information Bridge Server](#)

Citations per year



Conclusions



- Neutrino masses means that we need lepton flavour violation
- Radiative neutrino mass \rightarrow explaining the smallness of the neutrino masses
- Strongly constrained by cLFV processes. cLFV processes will soon be measured with even more increased precision!
- Studying the influence of cLFV new physics in other contexts can guide our study of (c)LFV in neutrino mass models.
- Don't forget those flavour anomaly model papers could help guide your future pheno studies

Thank you for listening!