Fermion pair radiation by accelerating classical systems

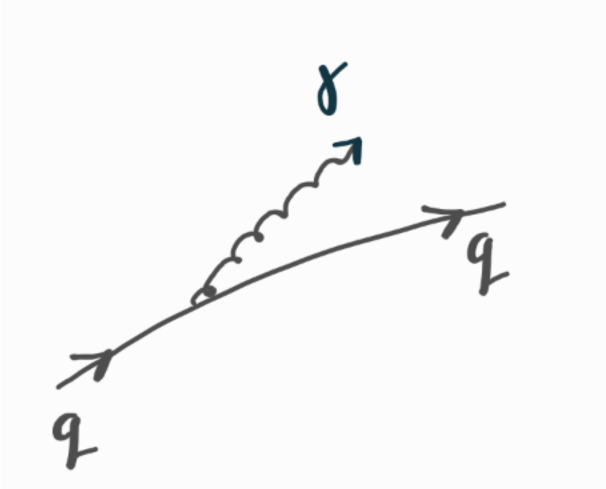
Margarita Gavrilova

Based on MG, Ghosh, Grossman, Tangarife, Tsai
arXiv:2301.01303

Chicago Workshop on Dark Matter and Neutrino Physics,
March 8, 2023

Cornell University

Larmor formula

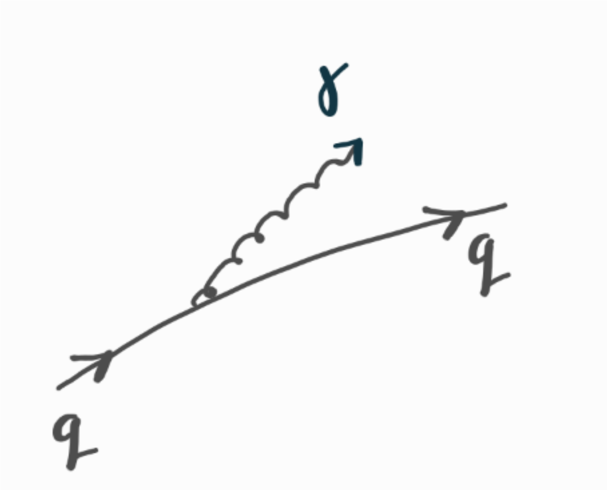


$$P_{\text{loss}} = \frac{1}{6\pi} q^2 \boldsymbol{a}^2$$

- Total power radiated by an accelerating non-relativitistic charge, where q is the particle's EM charge and $\boldsymbol{a}$ is its acceleration.

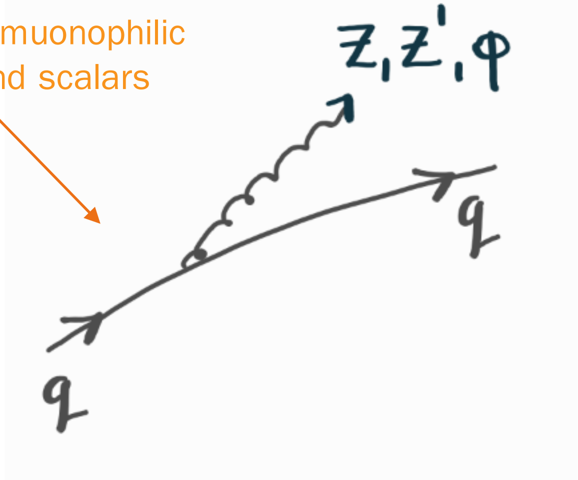
Generalizing the Larmor formula for exotic types of radiation

- radiation of massive bosons and scalars (SM or BSM)
- fermion pair radiation via massive bosons and scalars (SM or BSM)



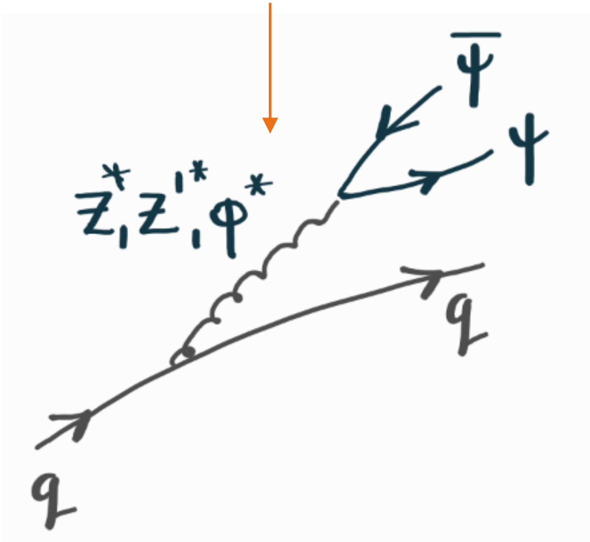
Larmor, 1897

ex. used to constrain **ultralight** muonophilic vectors and scalars



Krause, et al. Phys. Rev. D (1994)
Mohanty, Panda arXiv:9403205
Poddar, et al. arXiv:1906.00666, arXiv:1908.09732
Dror, et al. arXiv:1909.12845

Could this be used for some interesting pheno?



MG, Ghosh, Grossman, Tangarife, Tsai arXiv:2301.01303

Our goal: derive the generalization of the Larmor formula to the case of fermion pair radiation

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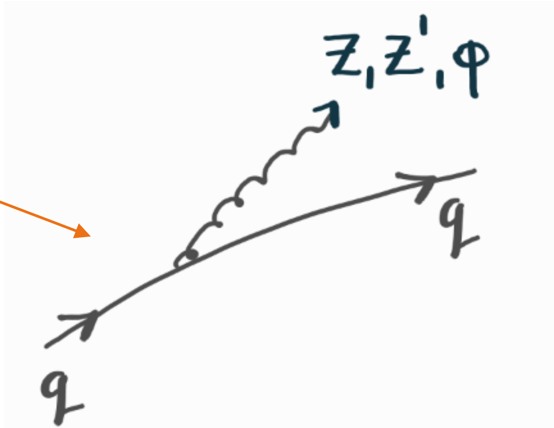
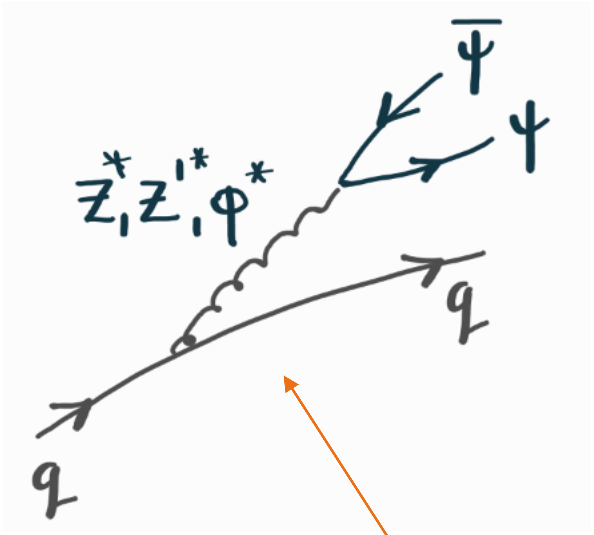
BUT WHY?
quickmeme.com

…but why?

- We are physicist and we like to **ask** and **answer** questions like this
  - one more example of the situation when a **fermion pair behaves like a boson**
  - other examples include Cooper pairs in superconductors, two fermion forces (see Mijo's talk Thu, 3pm)
  - can study **coherent fermion pair radiation**
- Potential of applying the result to **BSM searches** using astrophysical observations



has some interesting pheno applications

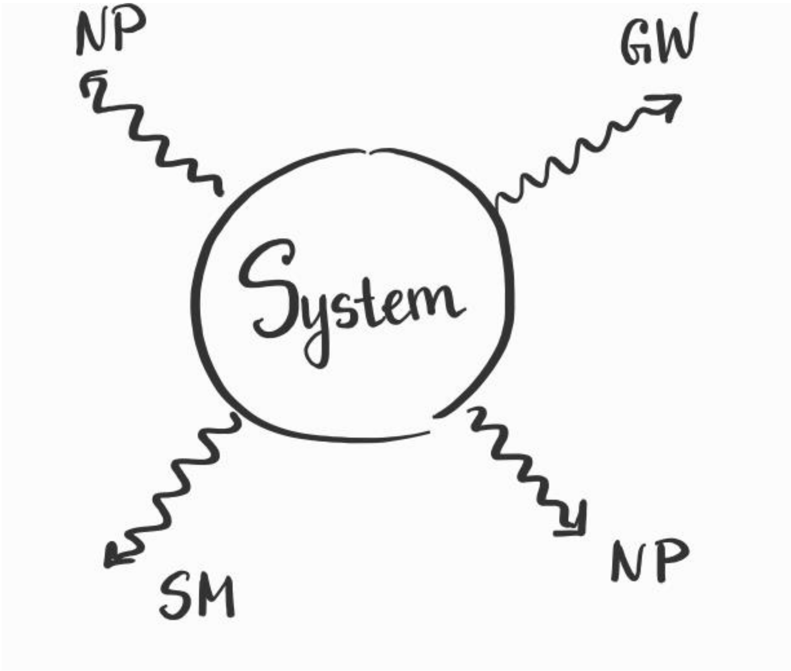


can provide a complimentary BSM test

Mandatory slide about the need for BSM

Image by Hubble Telescope
Credit: NASA, ESA, and M. Montes

The key idea



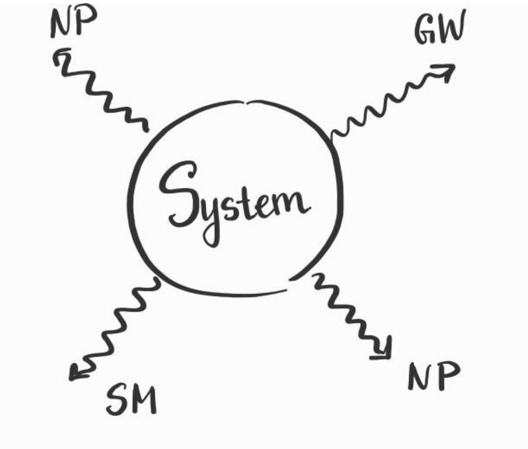
Consider a system for which we understand how it radiates away the energy via the SM radiation and via GW.

Direct or indirect measurement of the power radiated by the system

↓

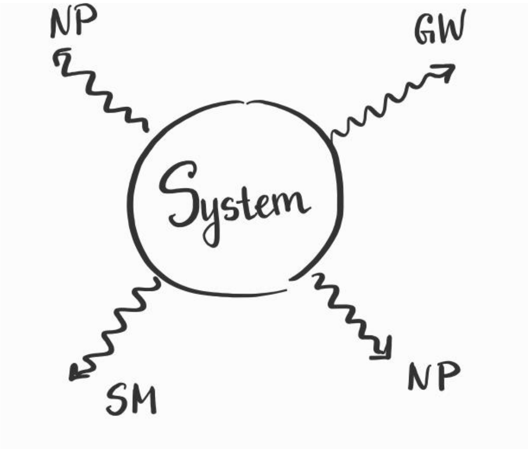
Test of the BSM physics

The key idea



Let us assume that the system has some characteristic frequency Ω_0 at which it radiates. In the case of the massive boson radiation, **what do you expect the exclusion curves look like in the mass-coupling plane?**

The key idea



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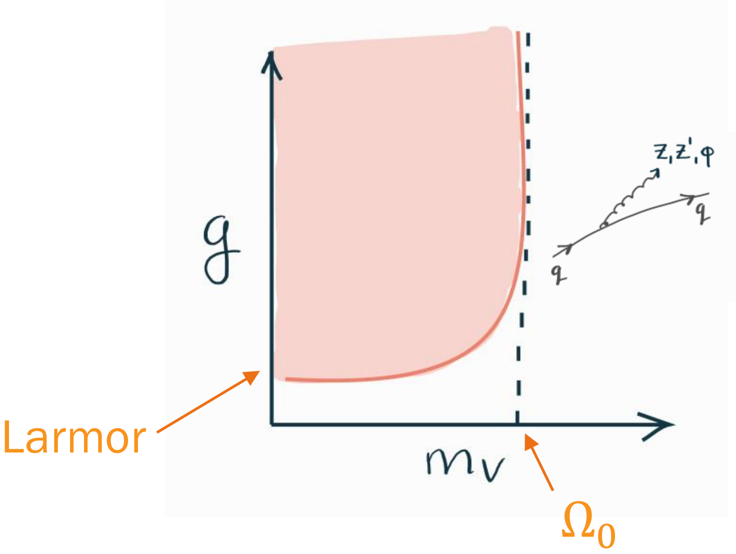
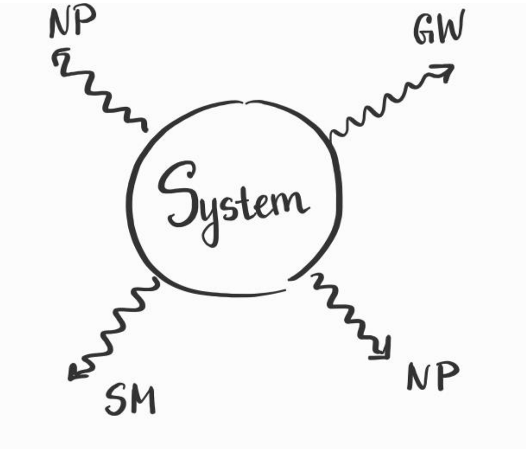


- in the limit $m_V \to 0$ we reproduce the Larmor formula
- at $m_V \to \Omega_0$ the power drops abruptly and thus there is no bound in the region $m_V > \Omega_0$

The key idea



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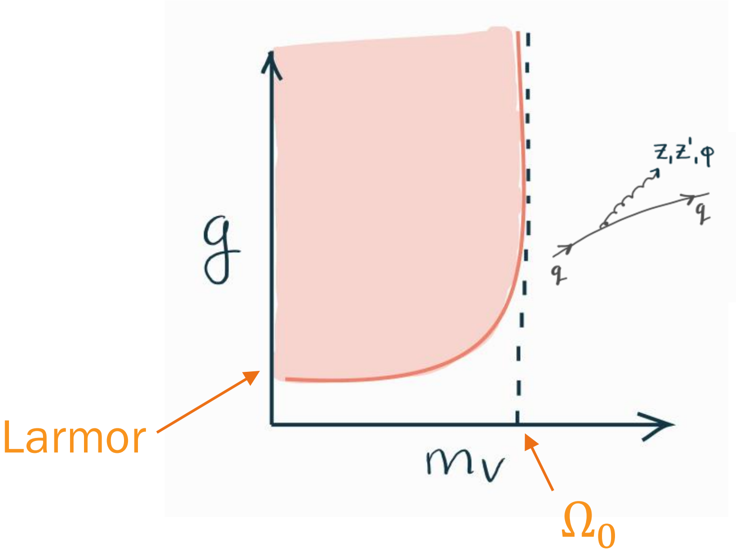

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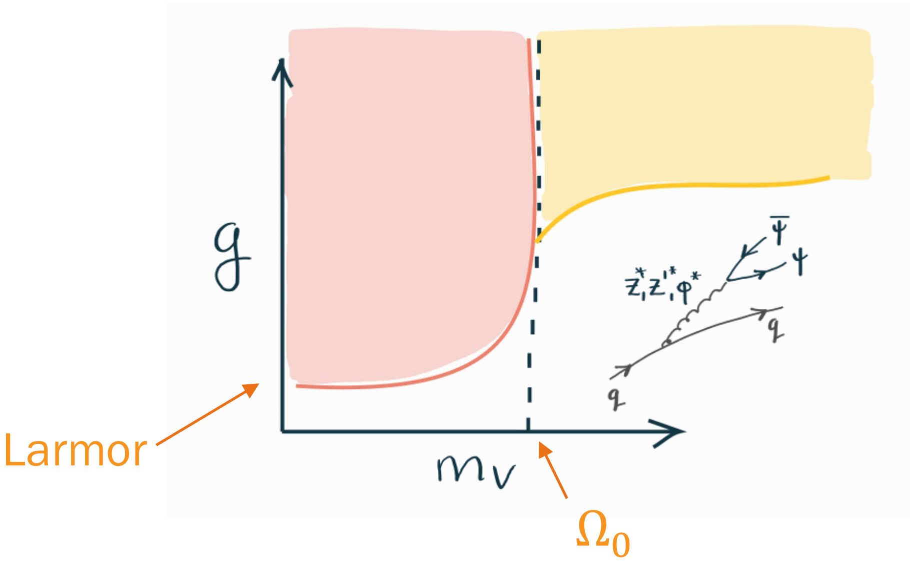


- in the limit $m_V \to 0$ we reproduce the Larmor formula
- at $m_V \to \Omega_0$ the power doesn't drop abruptly; the radiation of fermion pairs takes place via off-shell bosonic state giving us a new bound in the region $m_V > \Omega_0$

Two points to remember for now

1) We want to derive a power loss formula for the fermion pair radiation

2) We hope to apply this result to broaden the accessible parameter space in NP searches

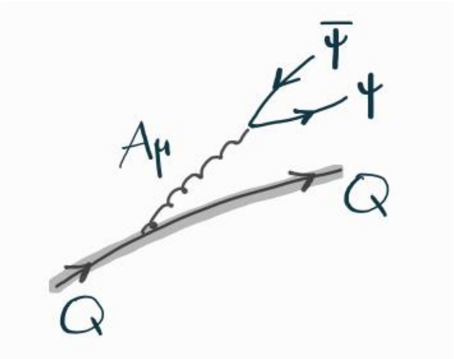
Plan

1) Fermion pair radiation by accelerating classical source
    a) derivation
    b) properties
2) Pulsar binaries as probes of NP
3) The case of the vector boson radiation
4) The case of the fermion pair radiation
5) Conclusions, caveats, questions

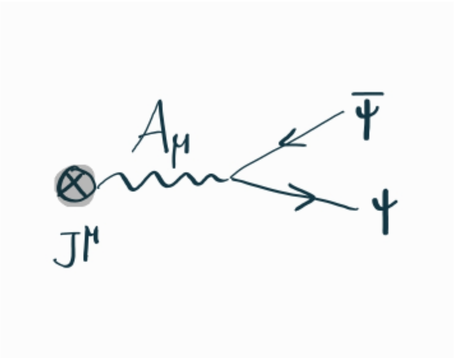
Fermion pair radiation: calculation

Setting up the problem

- Consider a **non-relativistic point-like object** with charge Q
- The point-like object **radiates fermion pairs**
- The radiation is realized via the object's **coupling to a massive boson**
- The boson is unstable and **decays into a fermion pair**
- Our goal is to calculate the power loss due to the fermion-pair radiation



Note, for simplicity, in what follows we focus on the case of the vector boson mediator. All the result are analogues in the case of the scalar mediator with $J^\mu(x) \to \rho(x)$ and $A_\mu \to \phi$



We describe the point-like object as a classical source using classical current

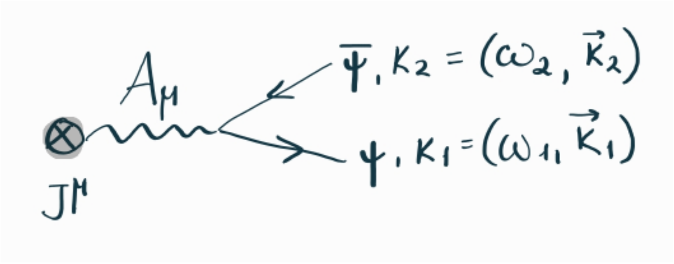
total charge of the source

$$\boxed{J^\mu_{\rm cl}(x) = Q\delta^3(\boldsymbol{x}-\boldsymbol{x}(t))u^\mu}$$

4-velocity

position of the point-like object

Calculation



The **power loss** due to the fermion pair radiation is calculated using

energies of the final state fermions

power loss

$$P_{\text{loss}} = \int (\omega_1 + \omega_2)\, \mathrm{d}\Gamma$$

differential rate of the fermion pair emission

In the case of the **periodic orbit**, the motion can be decomposed into harmonic modes with $\Omega_n = \mathrm{n}\Omega$. The total emission rate is then given by the sum of rates at different harmonics.

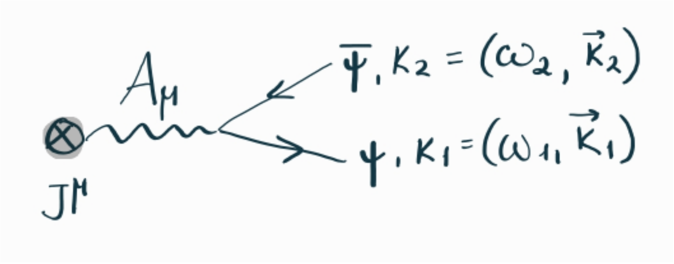
$$\mathrm{d}\Gamma = \sum_n \mathrm{d}\Gamma_n$$

emission rates at a given harmonic n, sum is over $n > 2m_\psi/\Omega$

$$\mathrm{d}\Gamma_n = \sum_{s_1, s_2} |\mathcal{M}_n(s_1, s_2)|^2 (2\pi)\delta(\Omega_n - \omega_1 - \omega_2) \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\pi)^3\omega_1} \frac{\mathrm{d}^3\boldsymbol{k}_2}{(2\pi)^3\omega_2}$$

$$P_{\text{loss}} = \sum P_n, \qquad P_n = \int (\omega_1 + \omega_2)\, \mathrm{d}\Gamma_n$$

Calculation (continued)



$$d\Gamma_n = \sum_{s_1,s_2} |\mathcal{M}_n(s_1,s_2)|^2 (2\pi)\delta(\Omega_n - \omega_1 - \omega_2) \frac{d^3\boldsymbol{k}_1}{(2\pi)^3\omega_1} \frac{d^3\boldsymbol{k}_2}{(2\pi)^3\omega_2}$$

To proceed, we need to specify

1) the microscopic physics that generates the fermion pair radiation (aka Lagrangian)
2) the trajectory of the point-like object (aka current)

We consider a vector mediator A_μ that corresponds to a broken $U(1)'$ and has mass m_A. The relevant terms in the effective Lagrangian are

$$\mathcal{L}_{\text{eff}} \supset g A_\mu J^\mu_{\text{cl}} + g q_\psi \bar{\psi} A_\mu \psi$$

recall that $J^\mu_{\text{cl}}(x) = Q\delta^3(\boldsymbol{x} - \boldsymbol{x}(t))u^\mu$

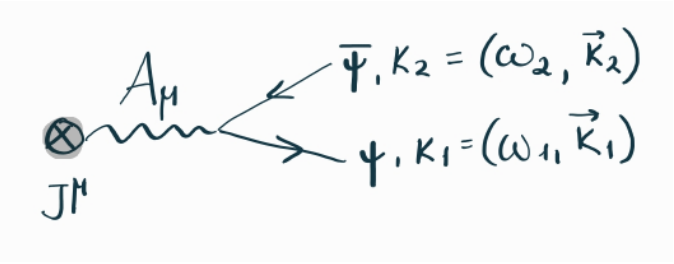
For the current, we consider elliptical motion of the point like object. Finally for the LO matrix element we have

Fourier transform of the current

$$\mathcal{M}_n(s_1,s_2) = g^2 q_\psi \bar{u}(k_1,s_1)\gamma^\mu v(k_2,s_2) \frac{i(-\eta_{\mu\nu} + (k_1+k_2)_\mu (k_1+k_2)_\nu / m_A^2)}{(k_1+k_2)^2 - m_A^2 + i m_A \Gamma_A} J^\nu_{\text{cl}}(\Omega_n)$$

decay rate

Calculation (continued)



$$P_{\text{loss}} = \sum P_n, \qquad P_n = \int (\omega_1 + \omega_2)\, \mathrm{d}\Gamma_n$$

$$\mathrm{d}\Gamma_n = \sum_{s_1, s_2} |\mathcal{M}_n(s_1, s_2)|^2 (2\pi)\delta(\Omega_n - \omega_1 - \omega_2) \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\pi)^3\omega_1} \frac{\mathrm{d}^3\boldsymbol{k}_2}{(2\pi)^3\omega_2}$$

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master formulas

These formulas allow us to calculate the power loss due to the fermion pair radiation by a point-like object. The fermion pair radiation is mediated by a massive gauge boson. The point-like object has charge Q under the corresponding symmetry. To proceed we need to:

1) calculate the Fourier transform $J^\mu(\Omega_n)$ for the case of an elliptical orbit
2) calculate the matrix element squared
3) calculate P_n for each harmonic (perform the 6D integration over the phase-space)
4) calculate P_{loss}

The power loss formula

a semi-major axis
Ω fundamental frequency
e eccentricity
m_A, Γ_A mass and decay rate of the boson
m_ψ mass of the fermion
$x = \omega_1/\Omega$

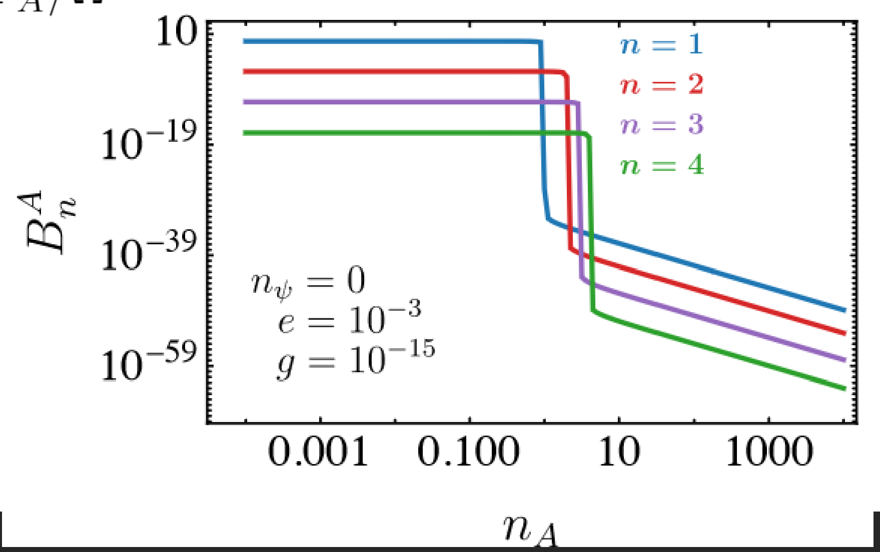
$$\boxed{P_n^A = \frac{g^4 q_\psi^2 Q^2}{12\pi^3} a^2 \Omega^4 \, B_n^A(n_A, n_\psi, n_\Gamma)}$$

$$B_n^A(n_A, n_\psi, n_\Gamma) \equiv \left(J_n'(ne)^2 + \frac{1-e^2}{e^2} J_n(ne)^2 \right) \int_{n_\psi}^{n-n_\psi} x \, F^A(x, n, n_A, n_\psi, n_\Gamma)$$

$$n_A \equiv m_A/\Omega, \qquad n_\psi \equiv m_\psi/\Omega, \qquad n_\Gamma \equiv \Gamma_A/\Omega$$

- $P_n^A \propto Q^2$, that is the fermion pair radiation is coherent
- Note the interplay of n_A and Ω_n
- Note the e dependence
- The power loss for the vector and scalar mediators has different functional form, but qualitatively behaves in the same way

Note, $m_\psi = 0$



Asymptotic behavior

We consider the case of the circular orbit ($e = 0$) and assume massless fermions ($m_\psi = 0$). In the limit of light mediators $m_A \ll \Omega$ ($n_A \ll 1$), we get

$$P^A(m_A \ll \Omega) \approx \frac{g^2}{6\pi} Q^2 a^2 \Omega^4 \longleftarrow$$

Recall that acceleration on a circular orbit is $a\Omega^2$

In the limit of heavy mediators $m_A \gg \Omega$ ($n_A \gg 1$), we get

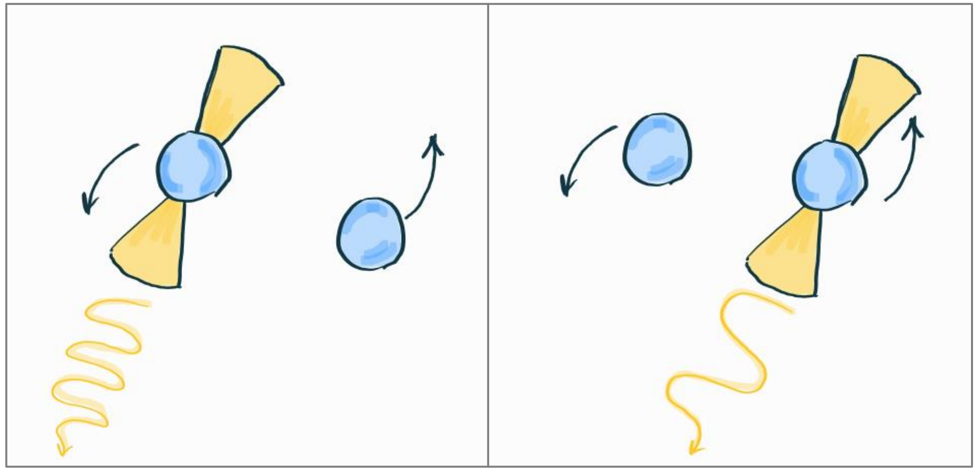
$$P^A(m_A \gg \Omega) \approx \frac{g^4 q_\psi^2 Q^2}{210\pi^3} \frac{a^2\Omega^8}{m_A^4} = \boxed{\frac{1}{35\pi^2} \frac{g^2 q_\psi^2 \Omega^4}{m_A^4}} \times P^A(m_A \ll \Omega)$$

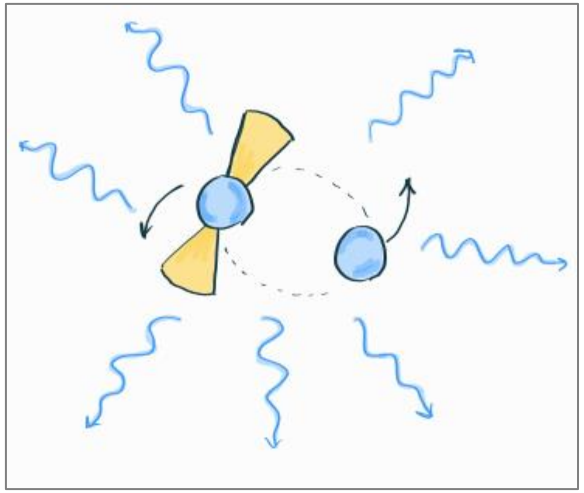
Suppression!

Pulsar binaries as NP probes

Pulsar binaries

- Pulsar (**pulsa**ting **r**adio source) = rotating NS which emits beams of EM radiation out of its poles
- Pulsar radiation can be observed only when it points towards the Earth → we detect **regular pulses** with a period of about $T_{pulse} \sim$ ms
- Pulsar binary is a system of a pulsar (NS) and a companion (often NS, WD)
- The frequency of the pulses is affected by the pulsar's motion
- Regular monitoring of arrival times of the pulsar signals allows to **track the pulsar's motion** and measure the orbit's eccentricity e, binary period T_b and the period decay $\boxed{\dot{T}_b}$





- NP effects: the decay of the orbit is accelerated
- Relevant scales: $T \sim 10^{-1} - 10^{3}$ days, $E \sim 10^{-23} - 10^{-19}$ eV

Pulsar binaries

For pulsar binaries measurement is typically within 1σ of the GW prediction

- Prior to 2015 (first direct GW detection by LIGO) binary pulsars were the only evidence for GW
- The measurements are extremely accurate with relative errors at percent or sub-percent level
- Period decay rate is directly related to the total power loss

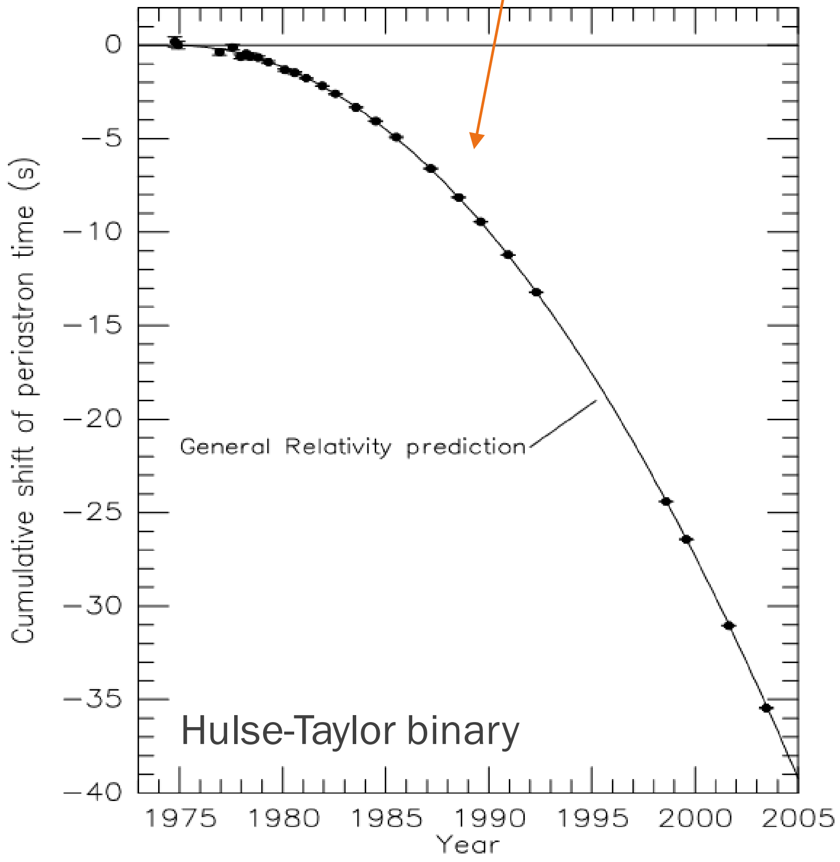
$$\dot{T}_b = \boxed{-6\pi a^{5/2} G_N^{-3/2} (m_1 m_2)^{-1} (m_1 + m_2)^{-1/2}} \times P_{\text{loss}}$$

parameters of the binary

power due to the GW radiation is also calculated with sub-percent to percent errors

$$P_{\text{loss}} = P_{\text{loss}}^{\text{GW}} + \boxed{P_{\text{loss}}^{\text{NP}}}$$

[place holder for your favorite BSM model]



Weisberg & Taylor arXiv:astro-ph/0407149

Applicability of the result

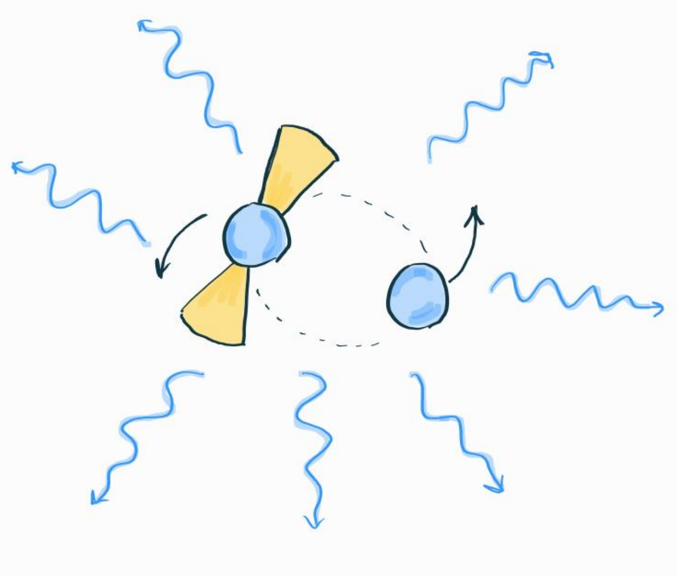
Typical parameters of pulsar binaries:

semi-major axis $a \sim 10^{24} - 10^{26}\ GeV^{-1}$

period $T \sim 10^{-1} - 10^{3}$days $\rightarrow \lambda \sim 10^{28} - 10^{32}\ GeV^{-1}$

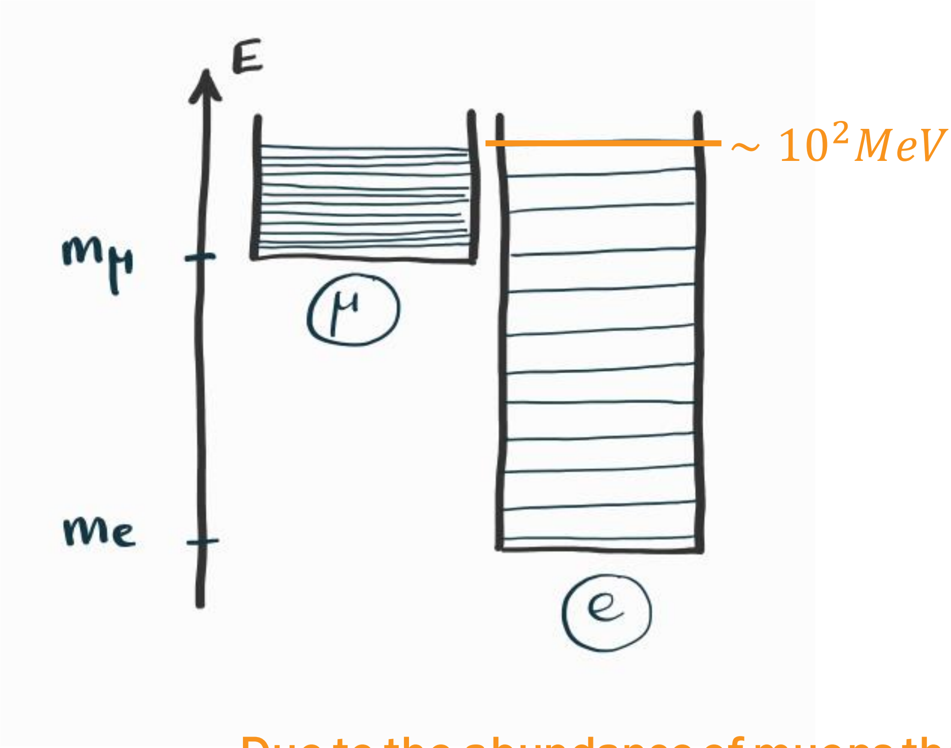
sizes of the stars $r_{NS} \sim 10km \sim 10^{19}\ GeV^{-1}$, $r_{WD} \sim 10^{3}km \sim 10^{21}\ GeV^{-1}$

- $\lambda \gg a$ (classical source)
- $r \ll a, \lambda$ (point like-objects & coherence)
- $v \sim a/T \sim 10^{-2}$ (non-relativistic)
- additionally, observed power loss is such that it has no significant effect on the eccentricity $e \approx const$



Our results for the fermion pair radiation can be applied to the case of pulsar binaries!

Neutron star content



- NS contain large numbers of muons $N(\mu) \sim 10^{55}$
- when $\mu_e > m_\mu$ it becomes favorable for electrons to decay into muons

$$e^- \to \mu^- + \bar{\nu}_\mu + \nu_e$$

- additionally, muonic beta-decay and inverse beta-decay become energetically favorable

$$n \to p + \mu^- + \bar{\nu}_\mu$$

$$p + \mu^- \to n + \nu_\mu$$

- muon decay is forbitten by Fermi statistics

$$\xcancel{\mu^- \to e^- + \bar{\nu}_e + \nu_\mu}$$

Due to the abundance of muons the effects of muonophilic physics get enhanced.
NS are unique laboratories to probe muonophilic new physics!

Concrete realization: $L_\mu - L_\tau$

- Additional $U(1)$ gauge symmetries with masses below the weak scale are simple extensions of the SM. Such gauge symmetries can act as mediators to the dark sector and appear in many BSM scenarios.
- The number of accidental conserved & anamaly-free $U(1)$ symmetries in the SM is limited → **there is a need to find as many experimental ways as possible to probe these light vectors**

$$\underbrace{B - L, \qquad L_e - L_\mu, \qquad L_e - L_\tau,}_{} \qquad \boxed{L_\mu - L_\tau}$$

strongly constrained by the 5th force searches, $g < 10^{-20}$

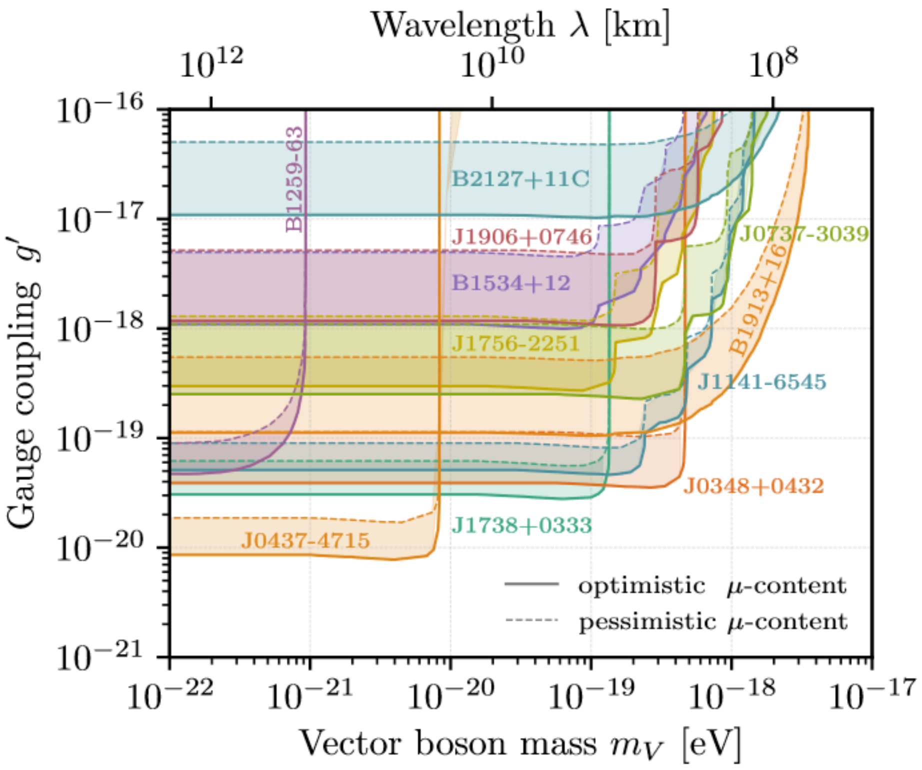
NOT bound by the 5th force constraints as the fractions of μ and τ in the ordinary matter are negligible

$$\boxed{\mathcal{L} \supset g A_\alpha \left(\bar{\mu}\gamma^\alpha\mu - \bar{\tau}\gamma^\alpha\tau + \bar{\nu}_\mu\gamma^\alpha\nu_\mu - \bar{\nu}_\tau\gamma^\alpha\nu_\tau\right)}$$

→ **Neutrino pair radiation by pulsar binaries**

Vector boson radiation

Constraints on $L_\mu - L_\tau$



Dror, Laha, Opferkuch, arXiv:1909.12845

- Relation between binary period decay and the power loss

$$\dot{T}_b = -6\pi a^{5/2} G_N^{-3/2} (m_1 m_2)^{-1} (m_1 + m_2)^{-1/2} \times P_{\text{loss}}$$

$\uparrow$ measured

$$P_{\text{loss}} = P_{\text{loss}}^{\text{GW}} + P_{\text{loss}}^{\text{NP}} \longleftarrow$$

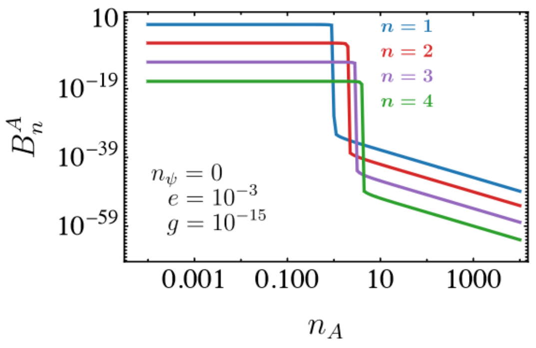
calculated as a function of g and m_A

- GW quadrupole radiation formula

$$P_{\text{loss}}^{GW} = \frac{32}{5} G \Omega^6 M^2 a^4 (1 - e^2)^{-7/2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)$$

- the resulting 2σ limits are plotted

Neutrino pair radiation



Neutrino pair radiation

One object of charge Q
on an elliptical orbit

$$P_n^A = \frac{g^4 q_\psi^2 Q^2}{12\pi^3}\, a^2 \Omega^4\, B_n^A(n_A, n_\psi, n_\Gamma)$$

$$B_n^M(n_M, n_\psi, n_\Gamma) \equiv \left(J_n'(ne)^2 + \frac{1-e^2}{e^2} J_n(ne)^2 \right) \int_{n_\psi}^{n-n_\psi} \mathrm{d}x\, F^M(x, n, n_M, n_\psi, n_\Gamma)$$

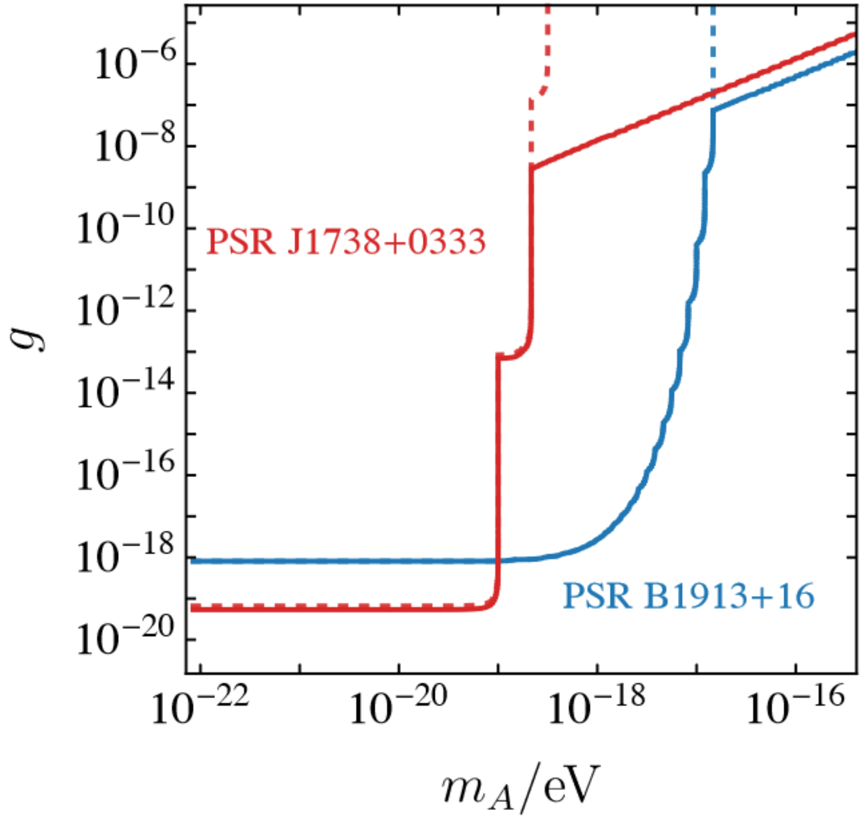
$$n_M \equiv m_M/\Omega, \qquad n_\psi \equiv m_\psi/\Omega, \qquad n_\Gamma \equiv \Gamma_M/\Omega$$

To generalize to the case of two objects on an elliptical orbit

$$J_{\mathrm{cl}}^\mu(x) = Q\delta^3(\boldsymbol{x} - \boldsymbol{x}(t))u^\mu \longrightarrow \boxed{J_{\mathrm{cl}}^\mu(x) = \sum_{i=1,2} Q_i \delta^3(\boldsymbol{x} - \boldsymbol{x}_i(t))u_i^\mu}$$

$$\boxed{P_n^A = \frac{g^4 q_\psi^2}{12\pi^3} M^2 \left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2} \right)^2 a^2 \Omega^4 B_n^A(n_A, n_\psi, n_\Gamma)}$$

Neutrino pair radiation



- Relation between binary period decay and the power loss

$$\dot{T}_b = -6\pi a^{5/2} G_N^{-3/2} (m_1 m_2)^{-1} (m_1 + m_2)^{-1/2} \times P_{\text{loss}}$$

↑

measured

$$P_{\text{loss}} = P_{\text{loss}}^{\text{GW}} + P_{\text{loss}}^{\text{NP}}$$

← calculated as a function of g and m_A

- GW quadrupole radiation formula

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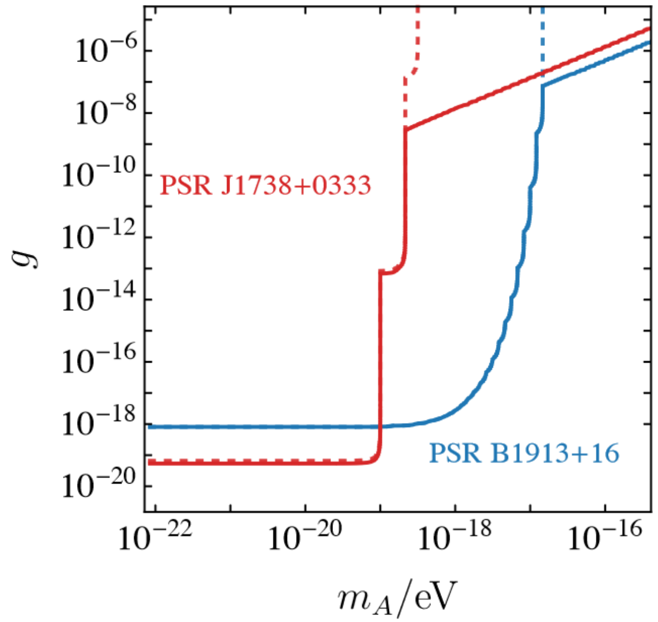
- the resulting 2σ limits are plotted

We talked about a gauge boson and neutrinos, but the results can be straightforwardly generalized to scalar mediators as well as boson pairs in the final state

Conclusions, caveats, questions

- You should think about this work as a **proof of principle:** fermion pair radiation can be used to probe a larger parameter space of BSM models
- BUT we are not yet there:
  - No comprehensive study of the bounds at large couplings was performed
  - The current muon number estimate breaks down at $g \sim 10^{-18}$
  - We require massless neutrinos

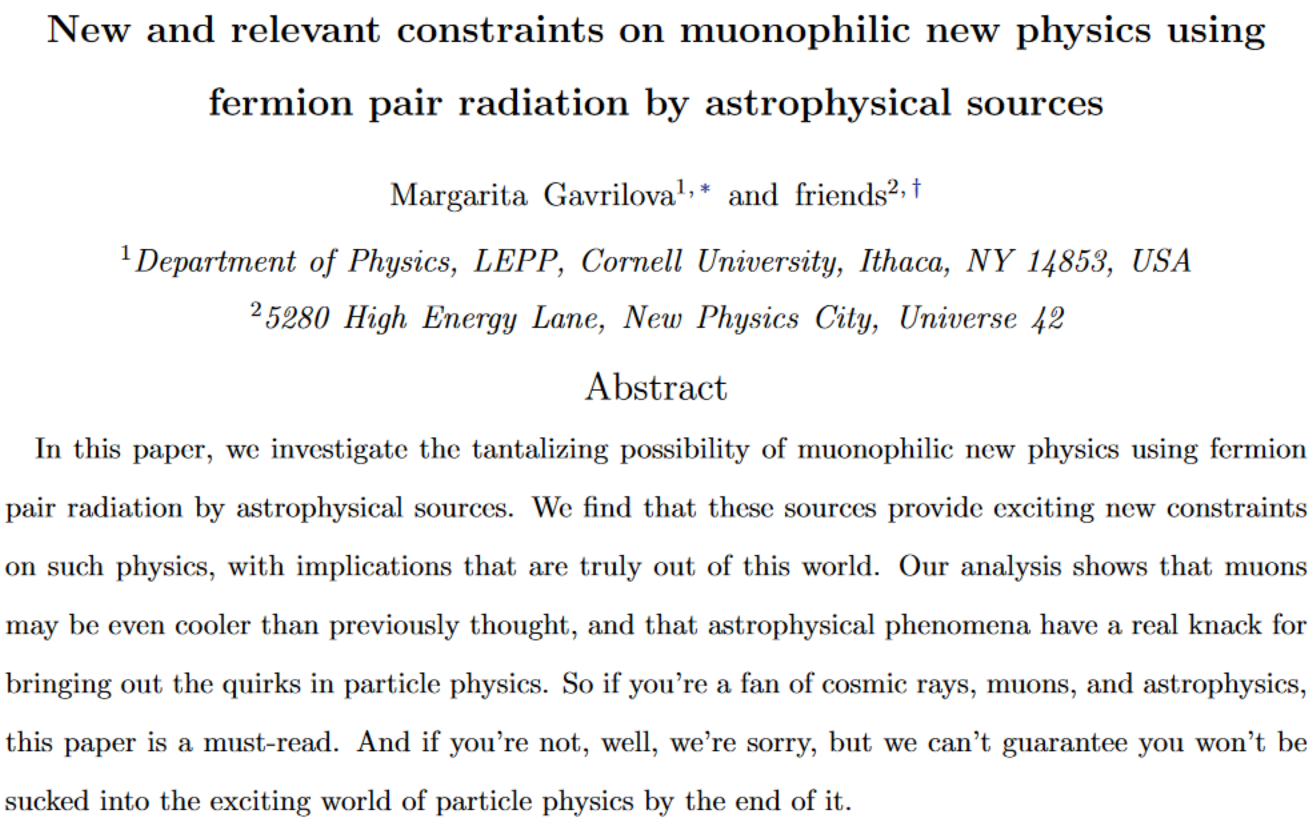


Call for input!

- Can we do smth about EoS? Can we get a better estimate of the muon number?
- Are there any other astro systems for which the fermion pair radiation can be relevant? Could we gain smth more practical by considering the fermion pair radiation for merger events? superradiance?
- Are there any cool BSM models which can be tested with our fermion pair radiation result?

Next paper

BE AWARE!
Abstract is generated by Chat GPT



New and relevant constraints on muonophilic new physics using fermion pair radiation by astrophysical sources

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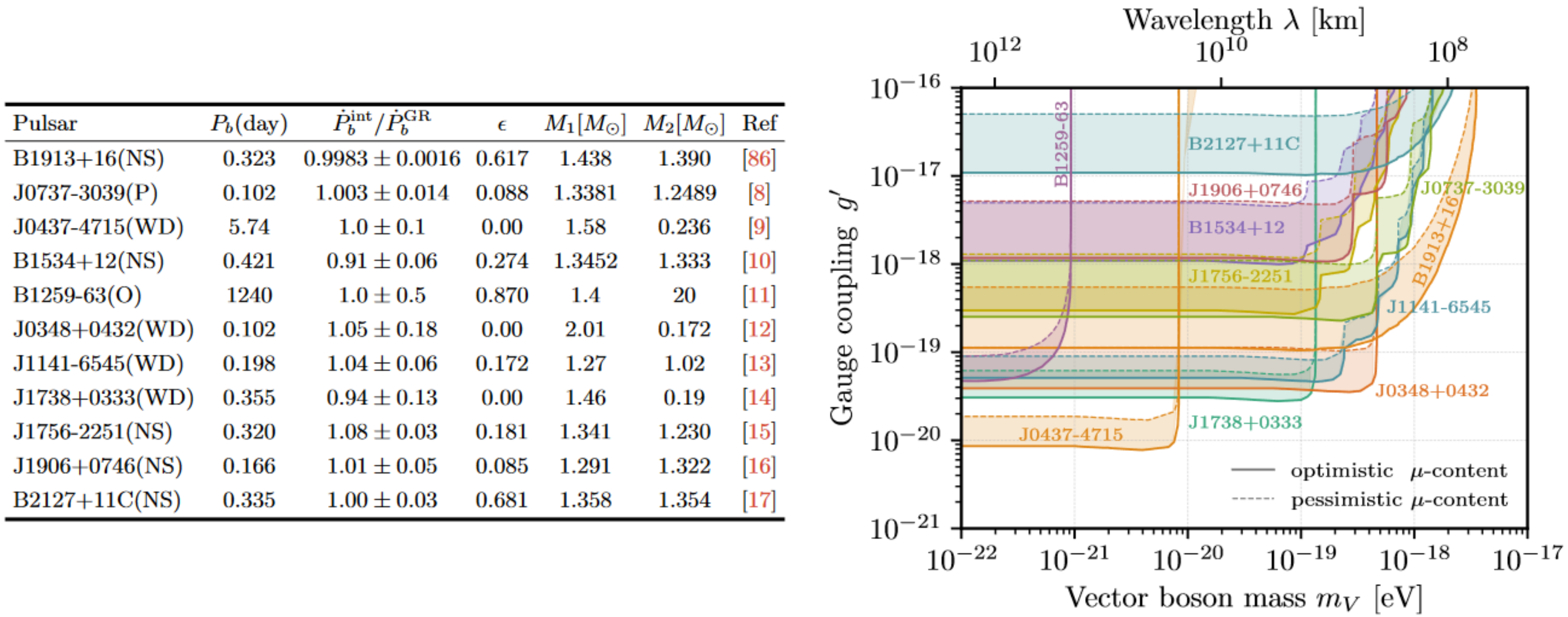
Abstract

In this paper, we investigate the tantalizing possibility of muonophilic new physics using fermion pair radiation by astrophysical sources. We find that these sources provide exciting new constraints on such physics, with implications that are truly out of this world. Our analysis shows that muons may be even cooler than previously thought, and that astrophysical phenomena have a real knack for bringing out the quirks in particle physics. So if you're a fan of cosmic rays, muons, and astrophysics, this paper is a must-read. And if you're not, well, we're sorry, but we can't guarantee you won't be sucked into the exciting world of particle physics by the end of it.

Backup

Dror, Laha, Opferkuch, arXiv:1909.12845



Pulsar	P_b(day)	$\dot{P}_b^{\rm int}/\dot{P}_b^{\rm GR}$	ϵ	$M_1[M_\odot]$	$M_2[M_\odot]$	Ref
B1913+16(NS)	0.323	0.9983 ± 0.0016	0.617	1.438	1.390	[86]
J0737-3039(P)	0.102	1.003 ± 0.014	0.088	1.3381	1.2489	[8]
J0437-4715(WD)	5.74	1.0 ± 0.1	0.00	1.58	0.236	[9]
B1534+12(NS)	0.421	0.91 ± 0.06	0.274	1.3452	1.333	[10]
B1259-63(O)	1240	1.0 ± 0.5	0.870	1.4	20	[11]
J0348+0432(WD)	0.102	1.05 ± 0.18	0.00	2.01	0.172	[12]
J1141-6545(WD)	0.198	1.04 ± 0.06	0.172	1.27	1.02	[13]
J1738+0333(WD)	0.355	0.94 ± 0.13	0.00	1.46	0.19	[14]
J1756-2251(NS)	0.320	1.08 ± 0.03	0.181	1.341	1.230	[15]
J1906+0746(NS)	0.166	1.01 ± 0.05	0.085	1.291	1.322	[16]
B2127+11C(NS)	0.335	1.00 ± 0.03	0.681	1.358	1.354	[17]

FIG. 5. Data used to set the constraints on gauged $L_\mu - L_\tau$ using pulsar binaries. **Left**: the parameters for each pulsar relevant for computing the constraints. The type of companion star is shown with the name, denoting a nonpulsating neutron star by NS, pulsar by P, white dwarf by WD, and O as an Oe-type star. **Right**: constraints from individual pulsars. For the parameter space above $g' = 10^{-18}$, several new effects as mentioned in Section V can become important.

Dror, Laha, Opferkuch, arXiv:1909.12845

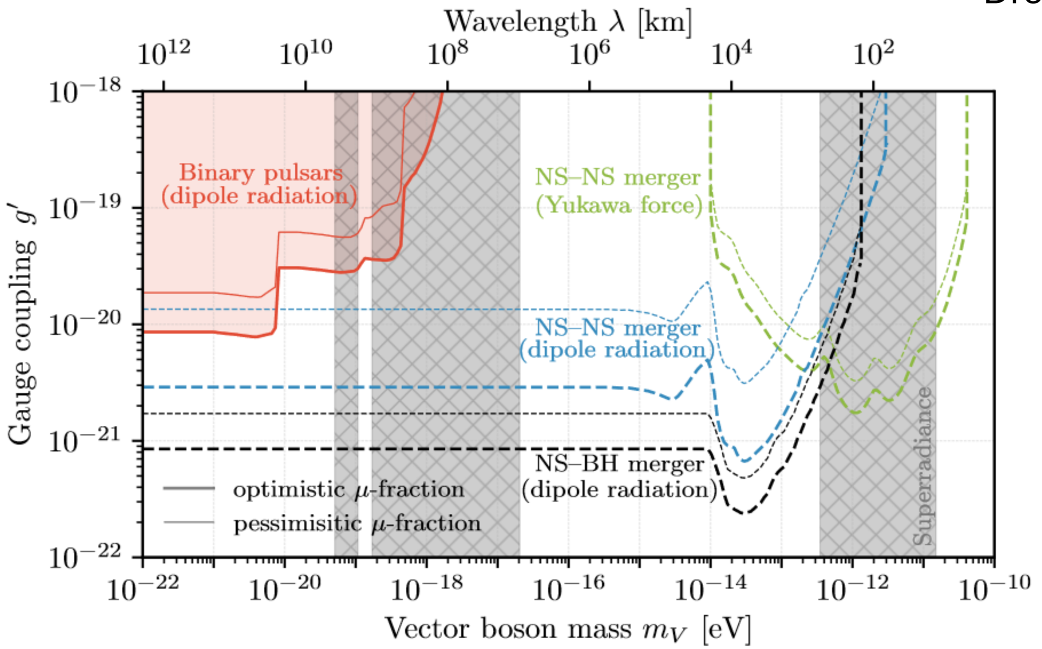


FIG. 2. Current sensitivity of NS binaries to a $L_\mu - L_\tau$ gauge coupling, g', as a function of the vector mass, m_V. The merger curves are projections and require a dedicated analysis to be carried out by the LIGO Collaboration. The gray hatched regions indicate parameter space where the light vector is constrained by BH superradiance considerations [33]. See Section V for the discussion of the boundaries of these constraints.