NEUTRINO EXCHANGE FORCES AND WHERE TO FIND THEM

The Chicago Neutrino and DM Workshop,
Loyola University Chicago,
March 09 2023

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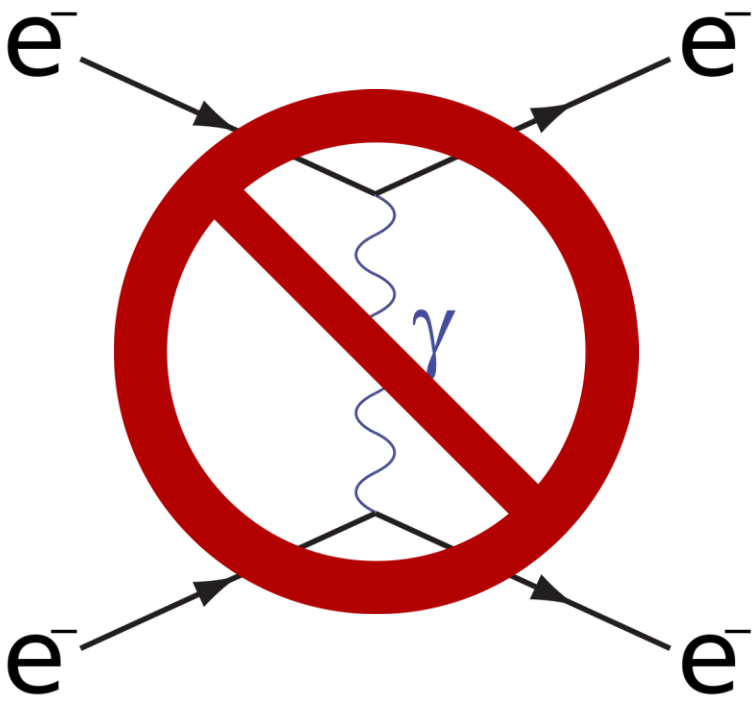
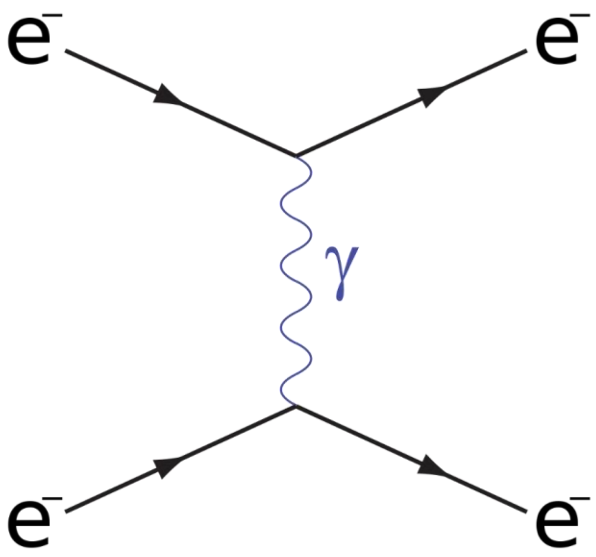
Cornell University

Without using any calculational tools of QFT:

Calculate the differential scattering cross section for electron-electron scattering.



THE COULOMB POTENTIAL



In the non-relativistic limit:

$$\mathcal{M} \sim \frac{1}{\mathbf{q}^2}$$

$$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \;\; \frac{1}{\mathbf{q}^2} \, e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{1}{r}$$

c.f. (Page 125 of Peskin)

FOR MASSIVE PROPAGATORS

Inverse range of the force

$$V(\mathbf{r}) \sim \int d^3\mathbf{q} \left(\frac{1}{\mathbf{q}^2 + m^2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-mr}}{r}$$

- Higher masses imply higher "virtuality" .
- Exponentially decaying potentials: The **probability of exchanging a virtual mass over larger distance falls exponentially with distance**. Falls faster when virtuality is high.
- More mathematically: the range of the force is given by the location of the branch cut in the matrix element in the t-plane.

STATIC POTENTIALS – A SUMMARY

In QFT, we have established machinery for computing scattering amplitudes. But..

In QM, scattering amplitudes are the Fourier Transforms of the interaction potential.

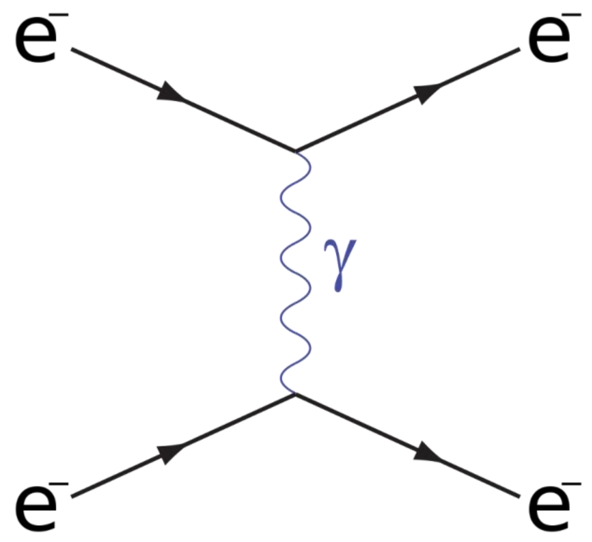
$$i\mathcal{M} \sim \int \mathrm{d}^3\mathbf{r} \;\; V(\mathbf{r})\; e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$V(\mathbf{r}) \sim i \int \mathrm{d}^3\mathbf{q} \;\; \mathcal{M}\; e^{-i\mathbf{q}\cdot\mathbf{r}}$$

We can compute the interaction potential from a scattering process by inverse Fourier Transformation of a **t-channel** diagram

CLASSICAL FORCE MEDIATION (TREE LEVEL)

Goal: Try to find a long-ranged force mediated by some fundamental particle

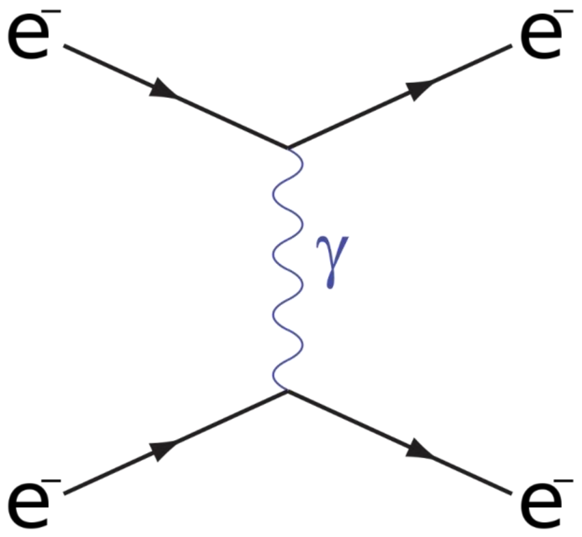


Who can do the job?

- A vector boson ✓
- A scalar ✓
- A fermion ✗

A single fermion exchanged cannot give us a potential as it alters the angular momentum state of the species involved. Potentials are classical.

QUANTUM FORCES (LOOPS)



Who can do the job?

- A scalar ✓
- A vector boson ✓
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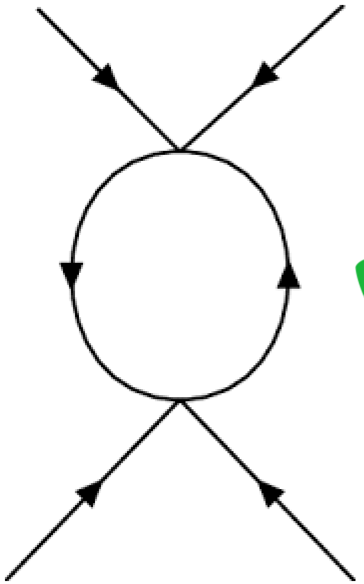
A single fermion exchanged cannot give us a potential as it alters the angular momentum state of the species involved. Potentials are classical.

Two Fermions?

Behave like bosons. Allowed!



✓

Figure stolen from the internet.
Arrows are wrong!

RANGE REQUIREMENTS

Short ranged forces not preferred for the sake of experiments

Medieval English Longbow, Range ~ 350 meters

Range of the weak force $\sim 10^{-18}$ meters

RANGE REQUIREMENTS

Short ranged forces not preferred for the sake of experiments

The Two-Neutrino Force!

Range $\sim 1\mu m$

RANGE REQUIREMENTS

M1, M2, M3 NEUTRINO SET

$52.99

Sorry, this item is out of stock

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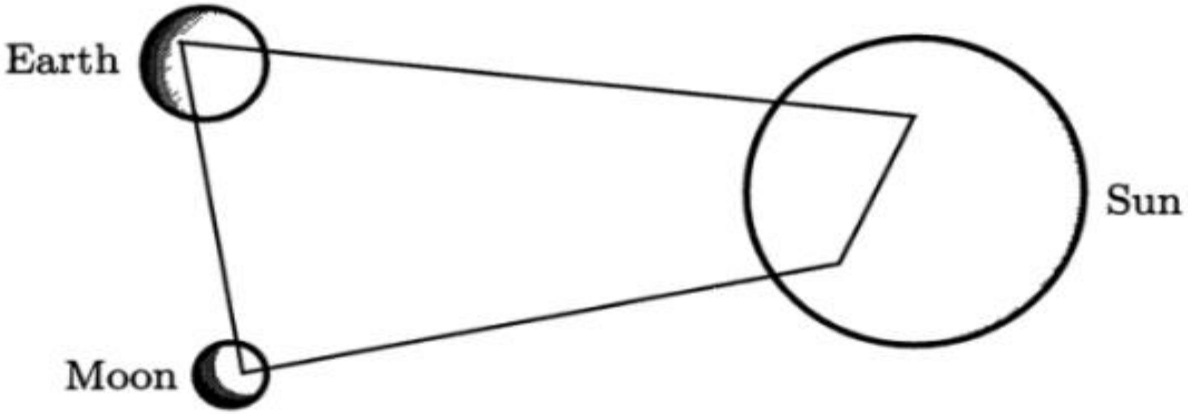
The neutrino-force as an explanation for gravity??

FEYNMAN LECTURES ON GRAVITATION

RICHARD P. FEYNMAN



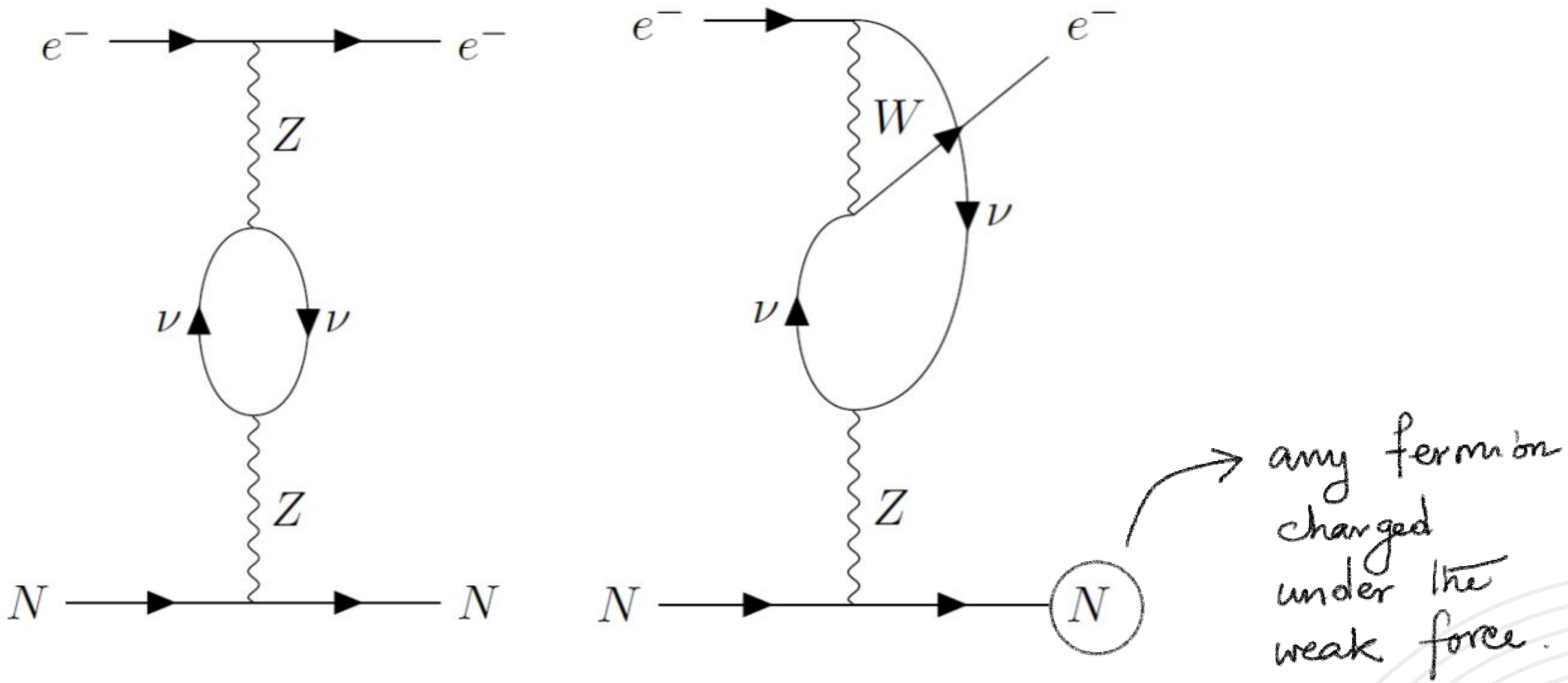
$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \qquad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

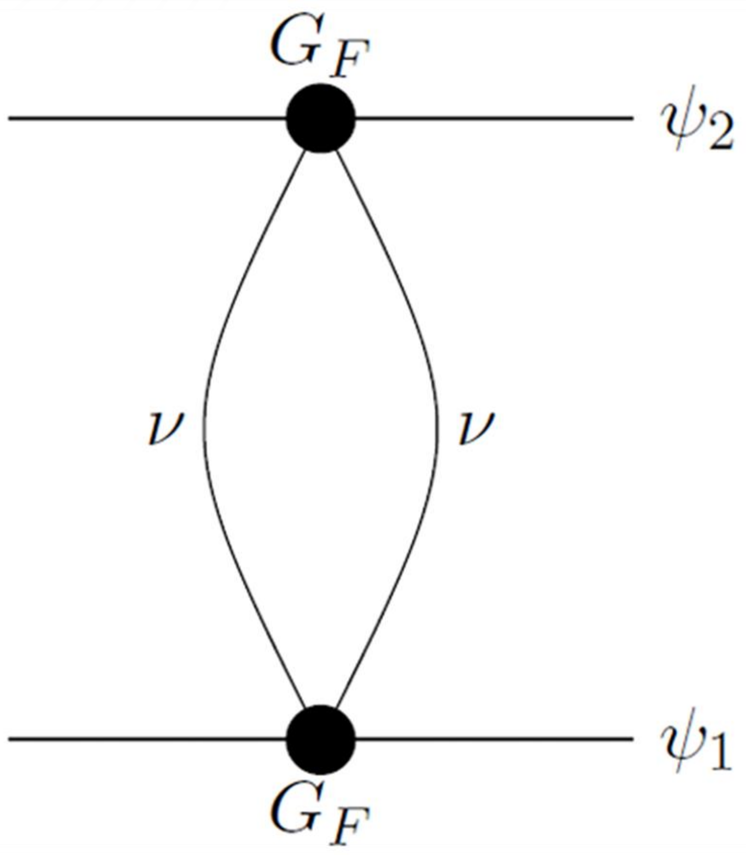
What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi\rho(R) R^2 dR}{2R^3}, \qquad (2.4.5)$$

THE 2-NEUTRINO FORCE in the SM



THE NEUTRINO FORCE IN THE 4-FERMI APPROXIMATION



At energy scales relevant to us, we can integrate out the weak bosons.

- For a long time, neutrinos were thought to be massless

$$V(r) = \frac{G_F^2}{4\pi^3 r^5}$$

Feinberg, Sucher, Au (~~1989~~) 60's
Sikivie, Hsu. (1994)

- Then mass was included, but three neutrino families and flavor mixing not accounted for

Grifols, Masso, Toldra (1996)

$$V_{\nu\nu}^{\text{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3}\frac{K_3(2m_\nu r)}{r^2}, \qquad V_{\nu\nu}^{\text{Majorana}}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3}\frac{K_2(2m_\nu r)}{r^3}$$

- Much weaker than gravity or electromagnetic forces at large distances.

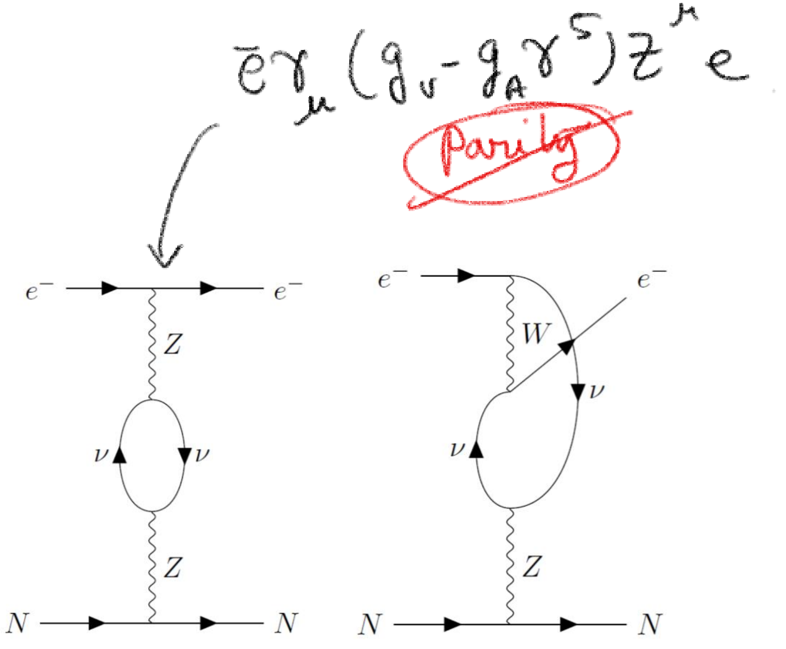
In 4-Fermi theory, this force is suppressed by **two powers of G_F.**

HOW TO SEE NEUTRINO FORCES IN ACTION

1. See if they break some symmetry

P-violation **because of electroweak vertices.**

MG, Grossman Y., Tangarife W. 1912.09444



2. Try to enhance the radial dependence them using backgrounds

Based on **MG, Grossman Y., Tangarife W., Xu X., Yu B, 2209.07082**

PLAN FOR TODAY

PART 1:

Atomic Parity Violation and the neutrino force

- How to see Parity Violation in atoms
- The neutrino force in atoms
- Applying results to Hydrogen – the simplest atomic system

Based on **MG, Grossman, Tangarife. arXiv: 1912.09444**

Based on **MG, Grossman, Tangarife, Xu ,Yu. arXiv: 2209.07082**

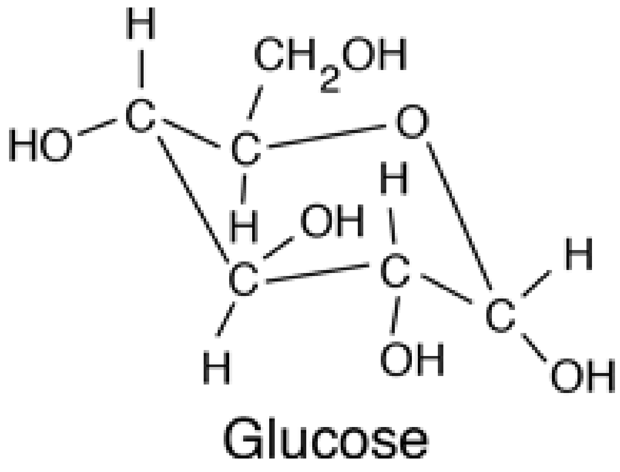
PART 2:

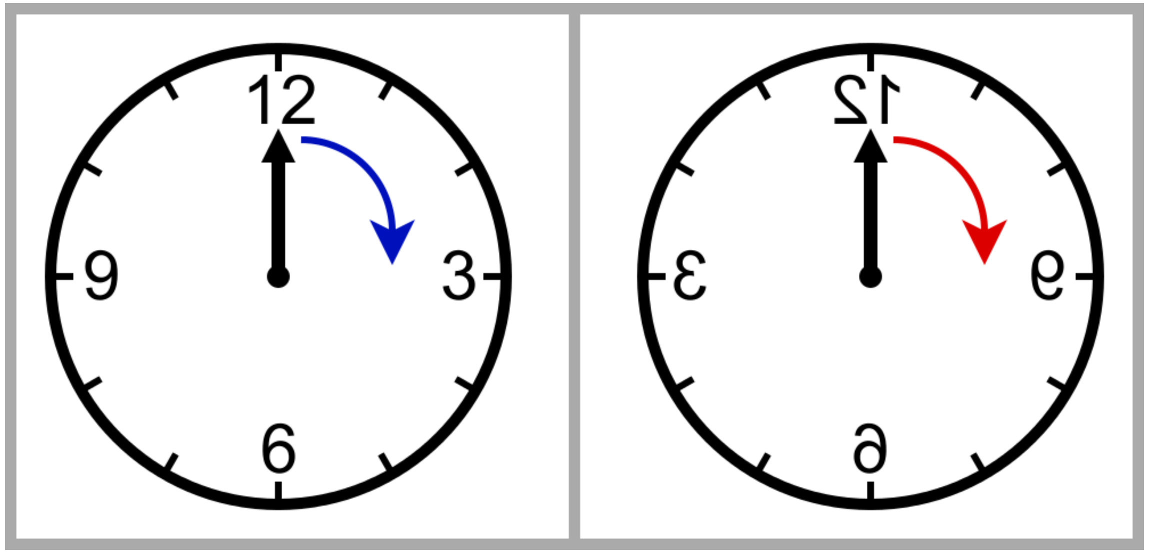
Enhanced neutrino forces in backgrounds

- How does a background affect the neutrino force?
- A simple calculation in a simple background
- Some technical problems


ATOMIC PARITY VIOLATION:
HOW NEUTRINOS TURN ATOMS SWEET...
(don’t worry, it’s still a physics talk!)

FINDING COMMON GROUND...

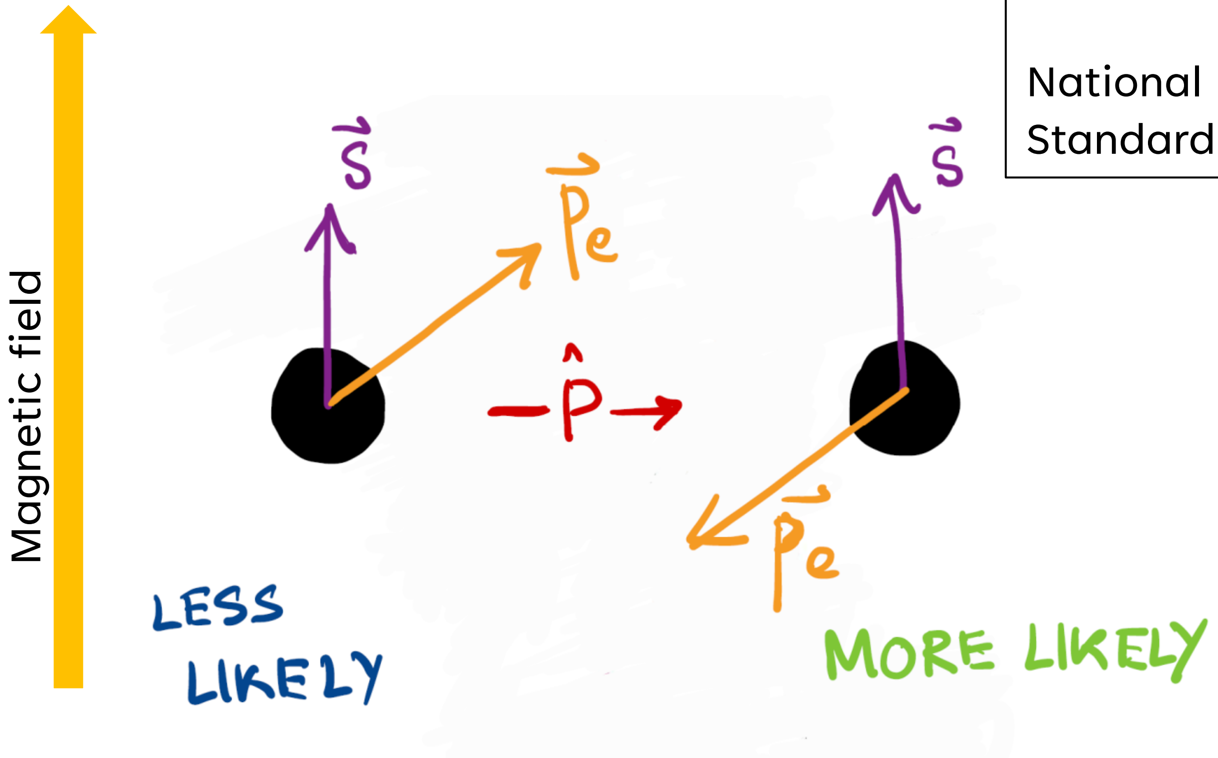




PARITY VIOLATION IN EXPERIMENTS

β-decay of Cobalt-60,

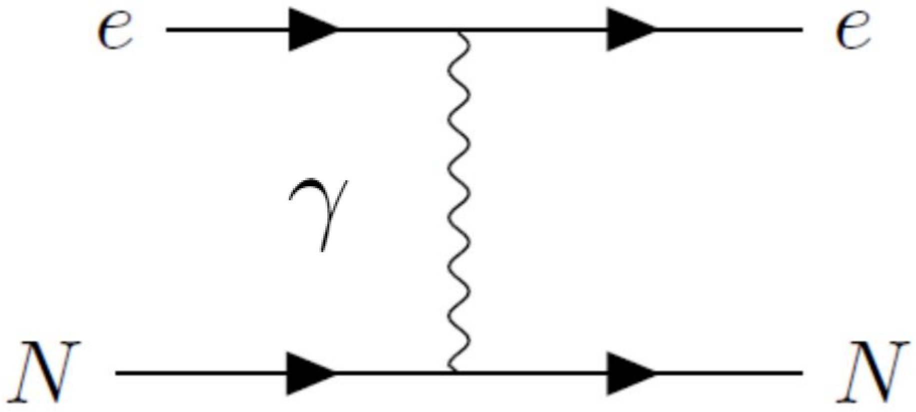
National Bureau of Standards, 1956



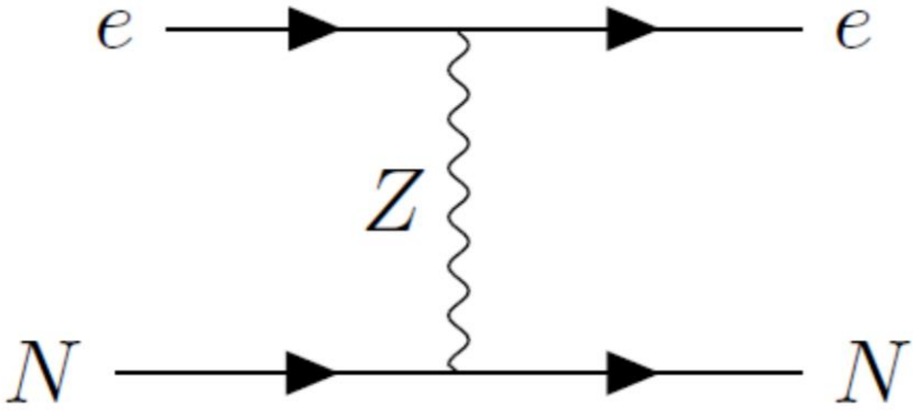
PARITY VIOLATION IN LOW ENERGY SYSTEMS?

TURN TO ATOMS AND MOLECULES (MOSTLY JUST ATOMS)

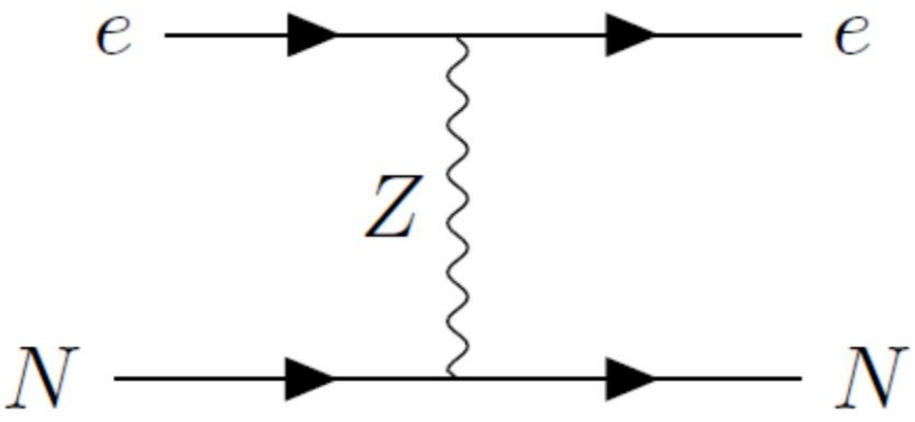
Interactions in atoms and molecules between electrons and nuclei are predominantly electromagnetic



But can include parity violating weak interactions as well…



THE TREE LEVEL PARITY VIOLATING POTENTIAL



$$V(\mathbf{r}) \sim \int d^3\mathbf{q} \left(\frac{1}{\mathbf{q}^2 + m_Z^2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-m_z r}}{r}$$

- The force is a Yukawa-force with range given by the inverse of the propagator mass. In this case, the range is $1/m_Z$.
- Because the Z is very massive, the range of this force is too small to be of significance in atomic systems.

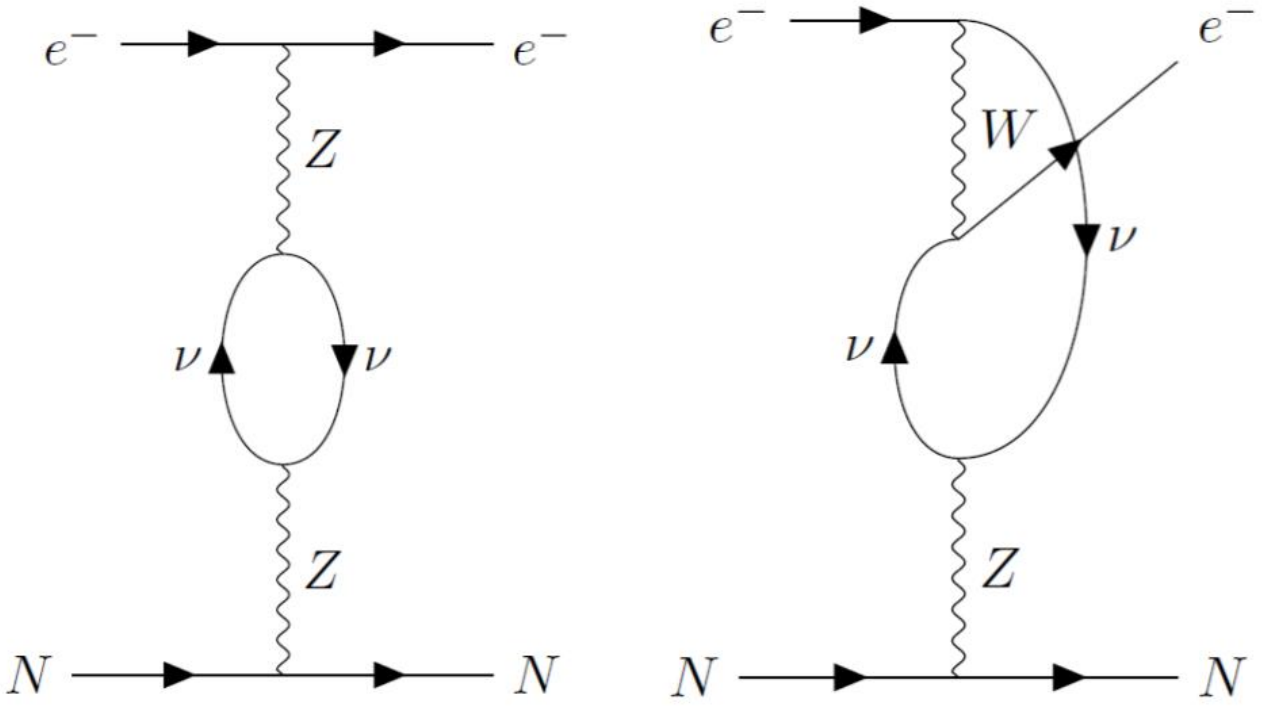
But this is not bad for us, as we'll see...

GOAL:

Find a long-ranged parity violating effect in atomic physics.

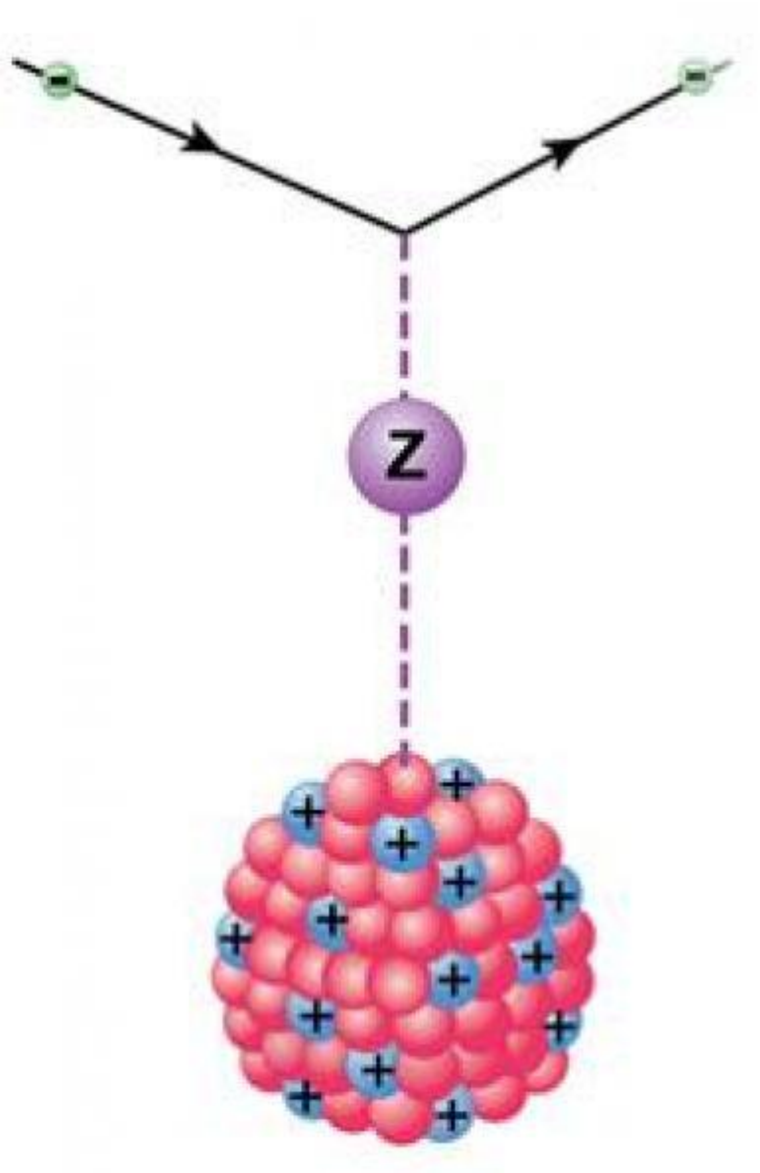
THE 2-NEUTRINO FORCE AT ONE LOOP

It Exists!



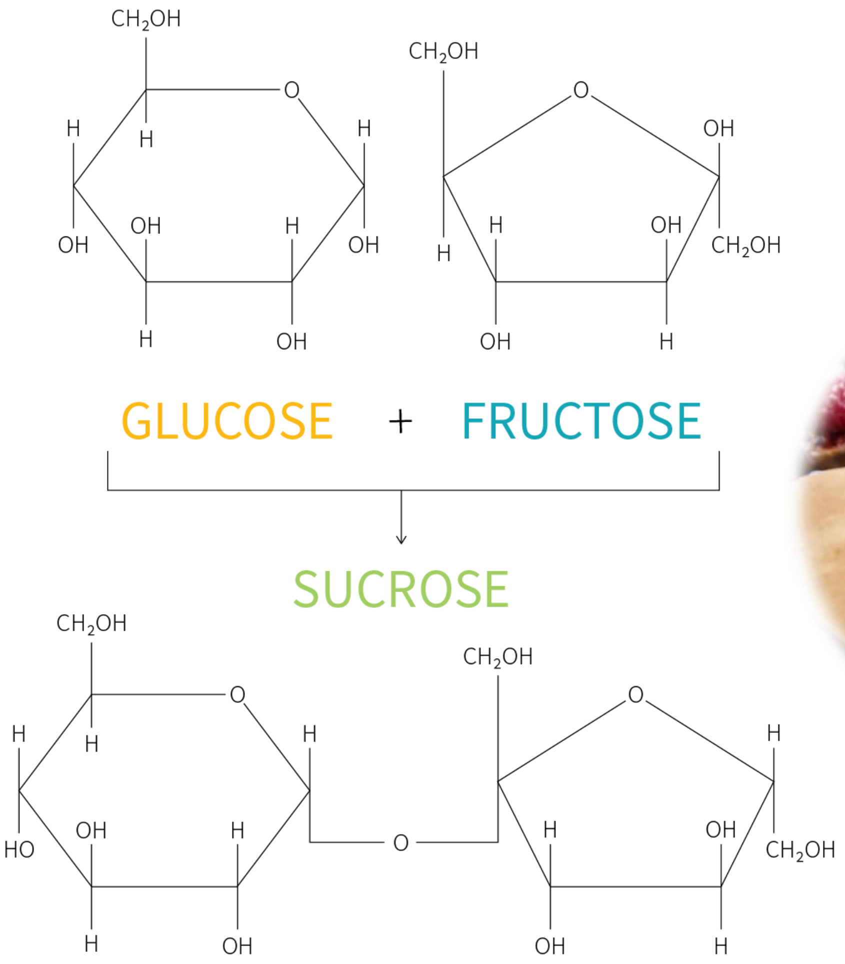
We expect a force that is long ranged – more effect on atomic electrons than the Z force

Said effect would be sensitive to the mass of the neutrinos



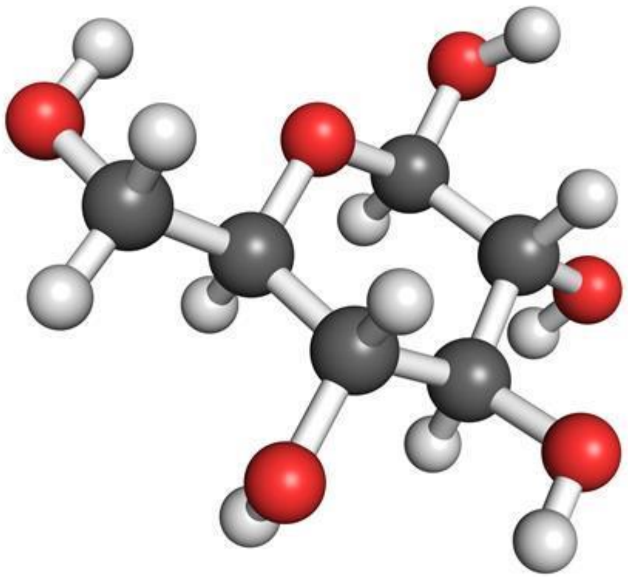
ATOMIC PARITY VIOLATION OBSERVABLES

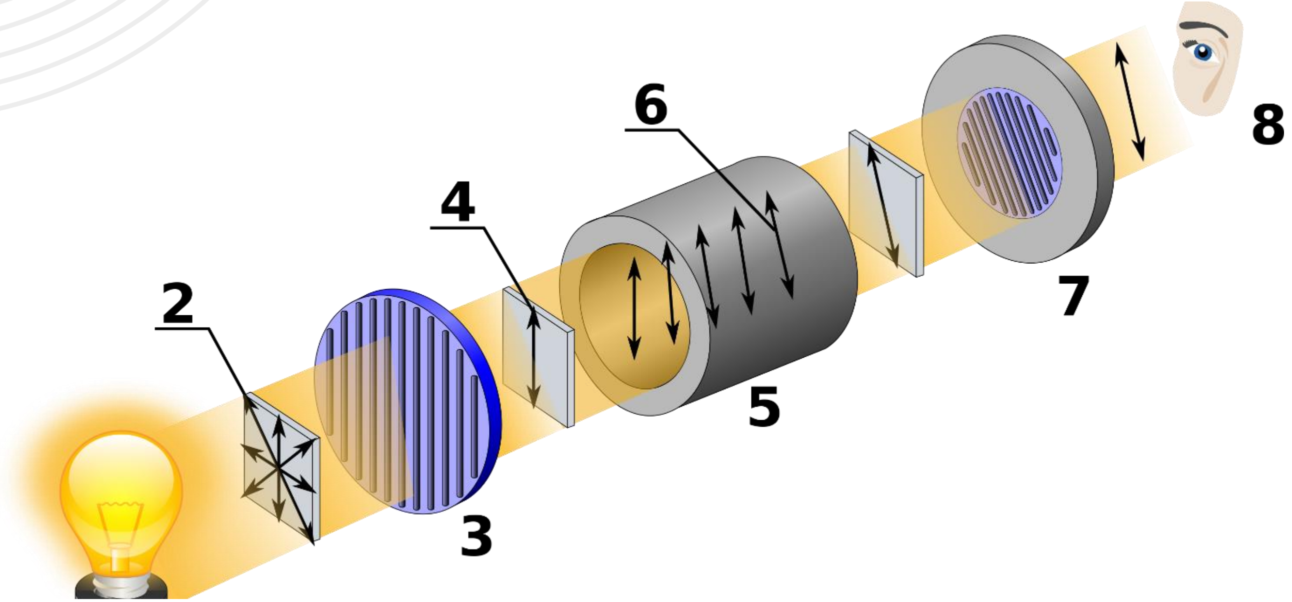
SOME LESSONS FROM SUGAR!

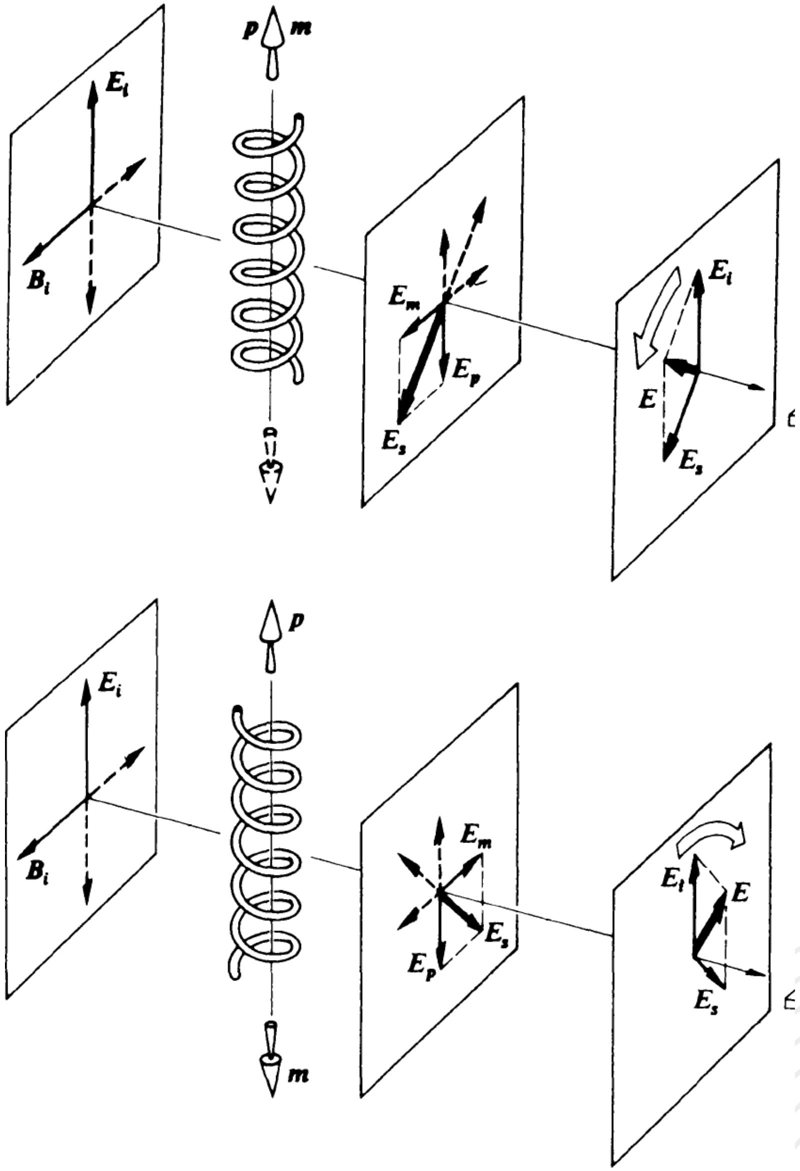


Real

Have a ~~virtual~~ cookie!







OPTICAL ACTIVITY IN SUGAR SOLUTION

THE REFRACTIVE INDEX

In optically active media:

Refractive indices for L-polarized right and R-polarized light are different.

Optical rotation is given by:

$$\Phi = \frac{\pi L}{\lambda} \mathcal{R}e(n_R(\lambda) - n_L(\lambda))$$

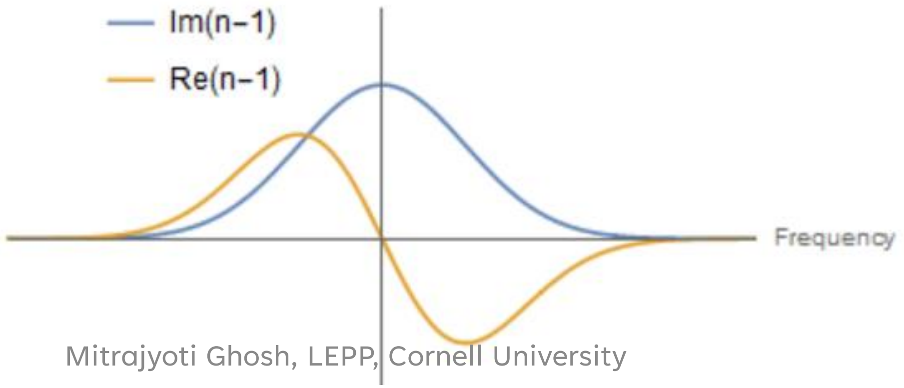
The refractive index is complex:

Real part: propagation, optical rotation

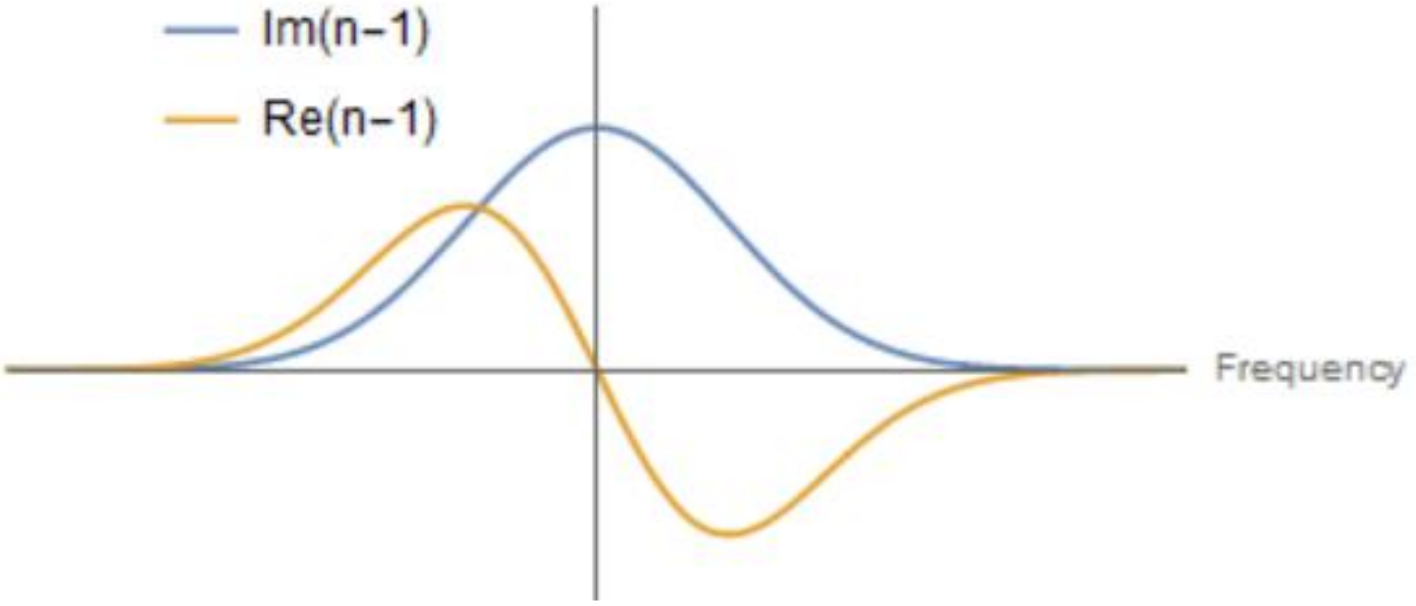
Imaginary part: absorption at resonance

The real and imaginary parts are related by the celebrated Kramers-Kronig relations.

$$\mathcal{R}e[n(\omega)] = 1 + \frac{2}{\pi} \int_0^\infty \mathrm{d}\omega' \, \frac{\omega' \, \mathcal{I}m[n(\omega')]}{\omega'^2 - \omega^2}$$



ROTATION NEAR RESONANCE



Resonance condition:

$$\omega_{\text{ext}} = \omega_1 - \omega_2$$

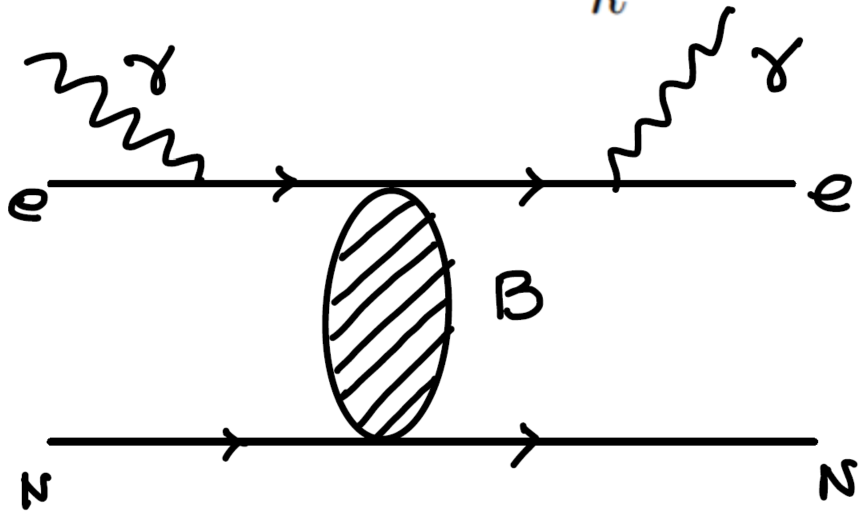
Any two eigen-energies of the system

Key Point: Expect optical rotation to be enhanced near a resonance

IN HEP LANGUAGE

The refractive index is related to the forward scattering amplitude of light

$$n_P^2(k) = 1 + \frac{4\pi N_e}{k^2} f_P(0).$$



If B = QED stuff,

$$f_L = f_R$$

No optical rotation.

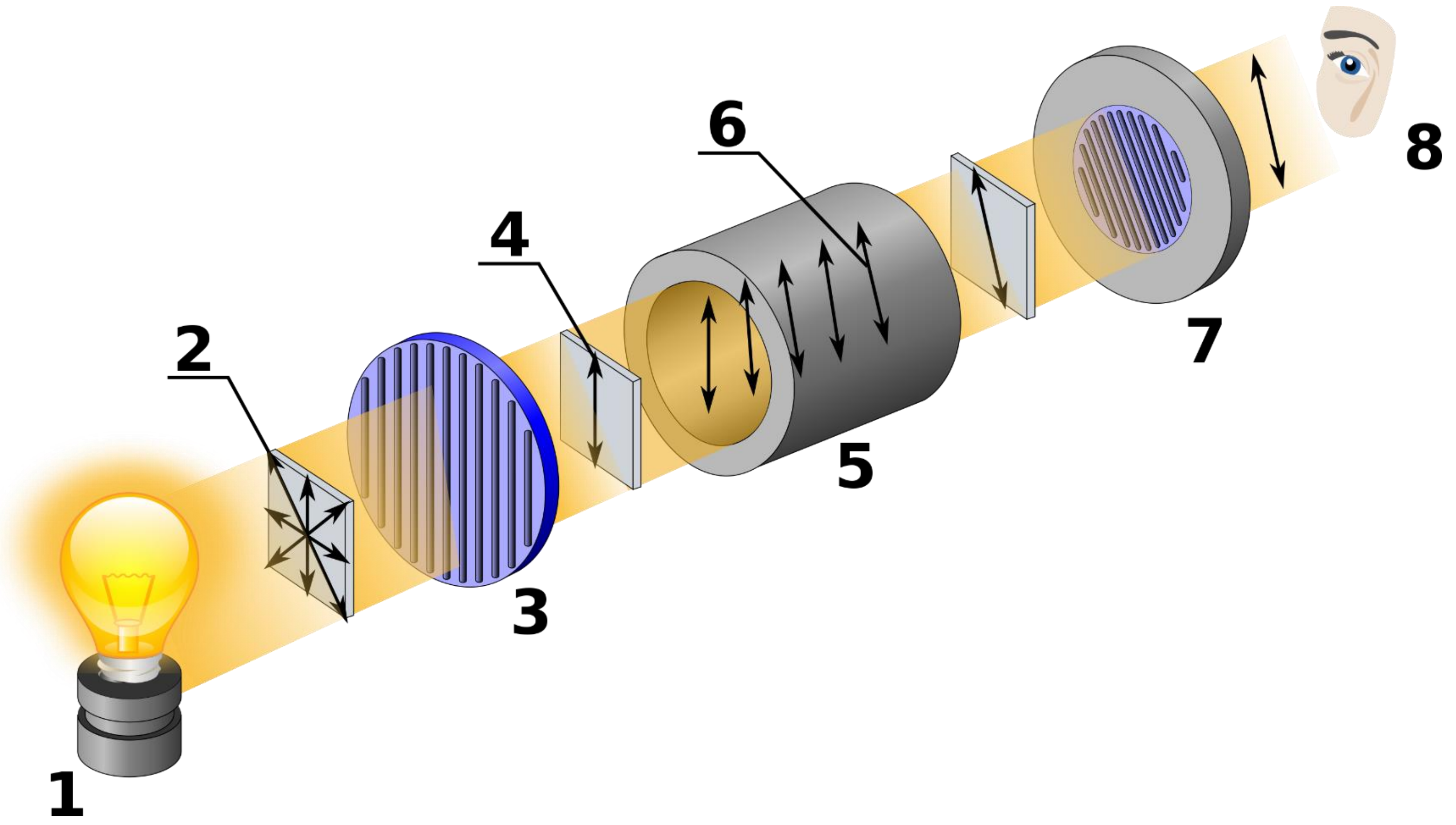
But if B = electroweak stuff,

$$f_L \neq f_R$$

Optical Activity!

HOW TO ‘~~SEE~~’ CALCULATE PARITY VIOLATION IN ATOMS?

The parity violation observable is therefore the angle of rotation of the plane of polarization of incident light.

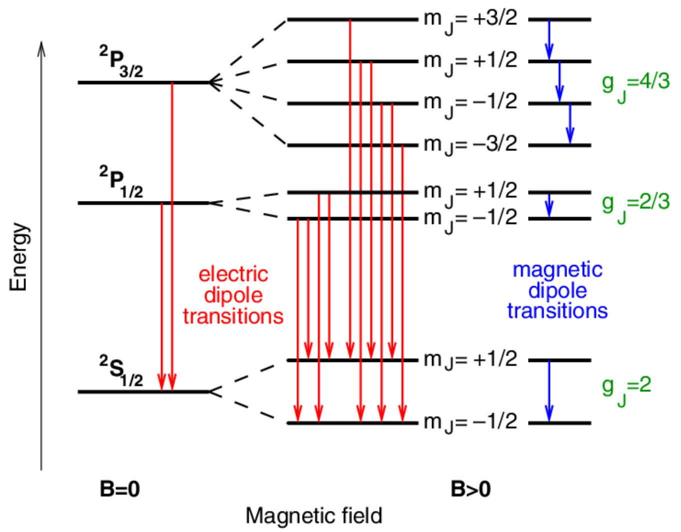


SHUT UP AND CALCULATE!

Instead of using QFT, one can use a QM workaround.

The strategy:

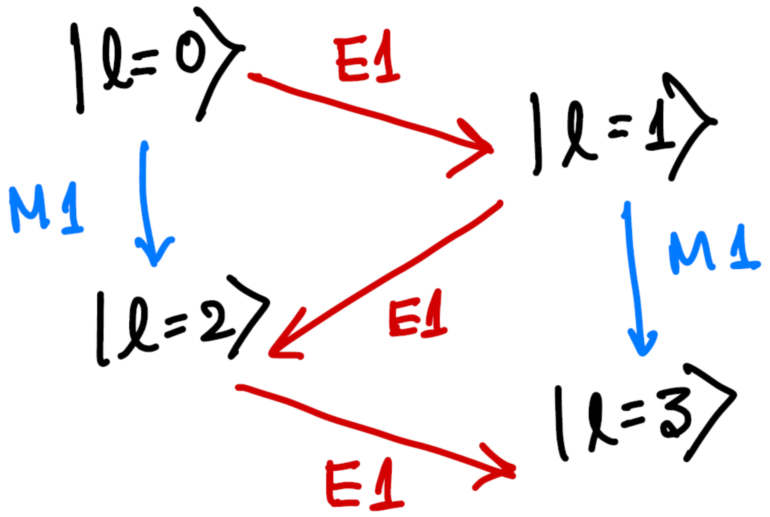
1. Consider a parity violating diagram
2. Compute a parity violating potential from it in the NR limit
3. **Perturb PC eigenstates to approximate true eigenstates**
4. Find **electromagnetic transition rates** (electric, magnetic dipole transitions) between true eigenstates as if the atom was immersed in an electromagnetic field and find the **absorption coefficients**.
5. **Relate absorption coefficients to refractive indices.**
6. Calculate optical rotation close to a resonance and you're done!



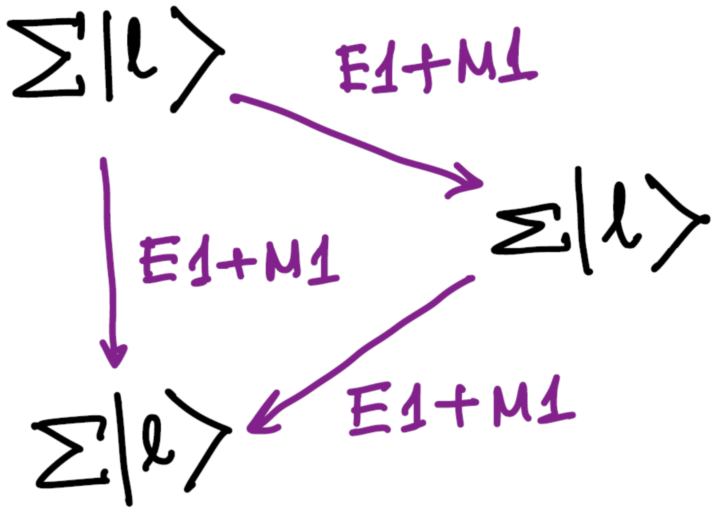
A PICTURE IS BETTER THAN A 1000 DENSE SLIDES

Parity Conserved

Note: Parity $= (-1)^{\ell}$



Parity Violated



Selection Rules Hold!

No selection rules anymore!

Prehistory (aka Pre-Covid Eternity)

AFTER SPENDING ~~ETERNITY~~ IN ATOMIC SPECTROSCOPY TEXTBOOKS

$$\Phi = \frac{4\pi L}{\lambda} \mathcal{R}e(n(\lambda) - 1)R$$

Average refractive index

$$R = \mathcal{I}m\left(\frac{E1_{PV}}{M1}\right)$$

Allowed only if parity is violated

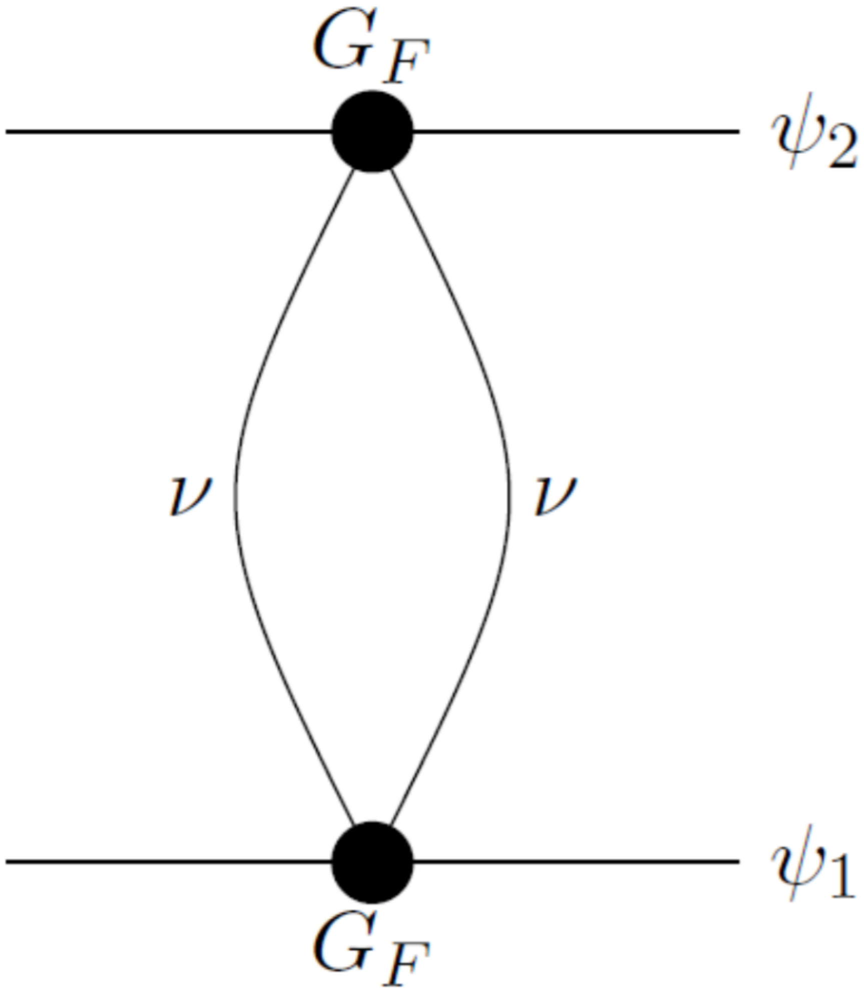
Allowed when parity is conserved

A REMARK:

If we choose a resonance where states are obtained by perturbing opposite parity eigenstates, then:

$$R \sim \mathcal{I}m\left(\frac{M1}{E1}\right)$$

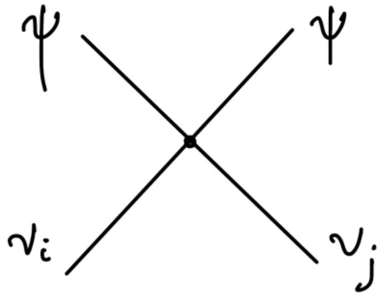
But M1≪E1, so doing this isn't worth it

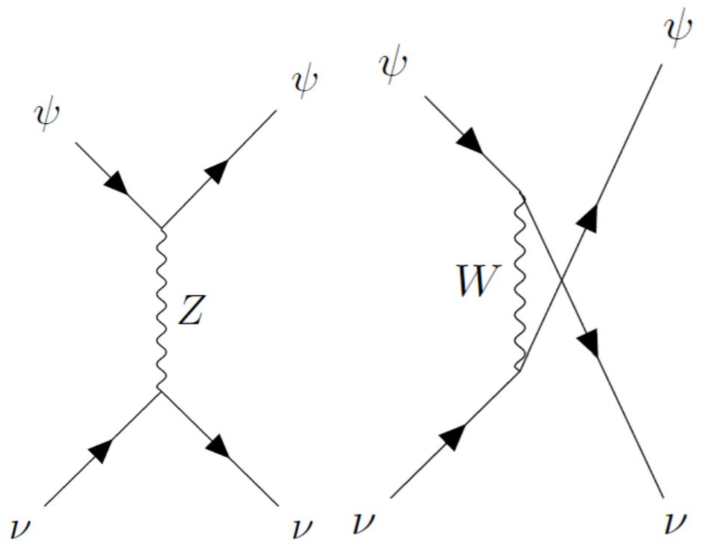


THE 2-NEUTRINO FORCE : Beyond $1/r^5$

Need spin and momentum dependent Parity Violating terms in the neutrino potential

THE 2-FERMION-2-NEUTRINO VERTEX





$$\begin{aligned}\mathcal{O}_{ij} &= (\mathcal{O}_Z)_{ij} + (\mathcal{O}_W)_{ij} \\ &= -\frac{G_F}{\sqrt{2}}\left[\bar{\psi}\gamma^\mu\{\delta_{ij}(g_V^\psi - g_A^\psi\gamma^5) + U_{\alpha j}U_{\alpha i}^*(1-\gamma^5)\}\psi\right]\left[\bar{\nu}_j\gamma_\mu(1-\gamma^5)\nu_i\right], \\ &= -\frac{G_F}{\sqrt{2}}\left[\bar{\psi}\gamma^\mu(a_{ij}^\psi - b_{ij}^\psi\gamma^5)\psi\right]\left[\bar{\nu}_j\gamma_\mu(1-\gamma^5)\nu_i\right].\end{aligned}$$

Vectorial coupling

Axial coupling

Technical

EFFECTIVE COUPLINGS

Technical

If ψ is a lepton

$$a_{ij}^{\psi} = \delta_{ij} g_V^{\psi} + U_{\alpha j} U_{\alpha i}^{*}$$

$$b_{ij}^{\psi} = \delta_{ij} g_A^{\psi} + U_{\alpha j} U_{\alpha i}^{*}$$

If ψ is not a lepton

$$a_{ij}^{\psi} = \delta_{ij} g_V^{\psi}$$

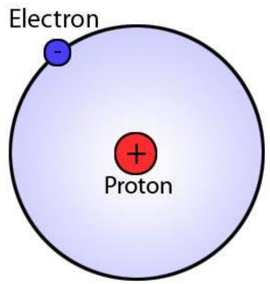
$$b_{ij}^{\psi} = \delta_{ij} g_A^{\psi}$$

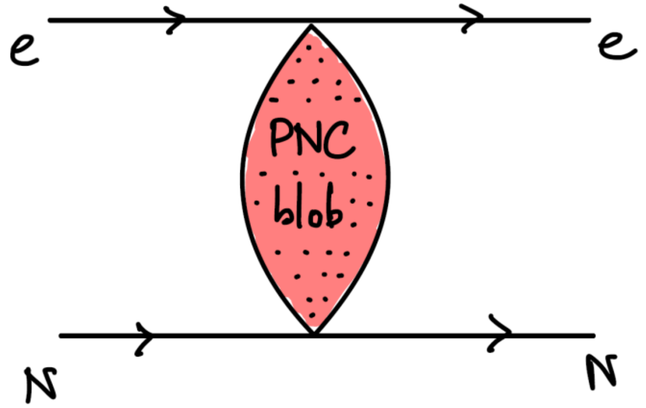
Generally, you would expect six diagrams for the force between two fermions. In atoms, the situation is simplified because one fermion is a non-lepton, and hence there are just three diagrams, one for each neutrino mass eigenstate in the loop.

GENERAL PARITY NON-CONSERVING POTENTIAL IN ATOMS

Assuming:

- A static nucleus, i.e, ignore effects $\sim m_e/m_N$.
- The non-relativistic limit and keep terms to linear order in velocity,





The general form of the PNC potential is:

$$V_{PNC}(r) = H_1 F(r)\vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r)\vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N) \cdot \vec{\nabla}\left[F(r)\right]$$

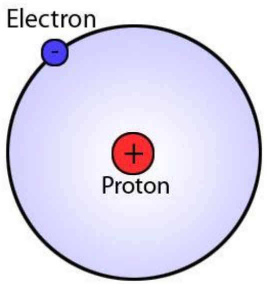
H for “helicity”

C for “Cross product”

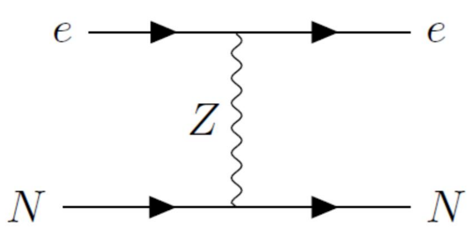
"AS SIMPLE AS POSSIBLE, BUT NO SIMPLER"

$$\psi_{nlm} = \langle r, \theta, \phi | nlm \rangle = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} e^{-r/na_0} \left[L_{n-l-1}^{2l+1}(2r/na_0)\right] Y_l^m(\theta, \phi).$$

The simplest atomic system, much loved and hated by undergrads and grads alike!



Not the most practical system to do experiments on, but good for theorists.



$$H_1 = H_1^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W} g_A^e g_V^p,$$

$$H_2 = H_2^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W} g_V^e g_A^p,$$

$$C = C^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W} \frac{g_V^e g_A^p}{2m_e},$$

$$F(r) = F^{\text{tree}}(r) = \frac{e^{-m_Z r}}{4\pi r}.$$

THE CASE OF HYDROGEN: THE Z-TREE

$$g_V^e = \left(-\frac{1}{2} + 2\sin^2\theta_W\right), \quad g_A^e = -\frac{1}{2}, \quad g_V^p = \left(\frac{1}{2} - 2\sin^2\theta_W\right), \quad g_A^p = \frac{G_A}{2}$$

Axial proton form factor

THE CASE OF HYDROGEN: ν-LOOP

We considered Dirac neutrinos. Majorana neutrinos left to a future work.

$$H_{1i} = H_{1i}^{\text{loop}} = -2\frac{a_i^p b_i^e}{m_e},$$

$$H_{2i} = H_{2i}^{\text{loop}} = 2\frac{a_i^e b_i^p}{m_e},$$

$$C_i = C_i^{\text{loop}} = \left(\frac{a_i^e b_i^p}{m_e} + \frac{a_i^p b_i^e}{m_p}\right)$$

$$F_i = F_i^{\text{loop}}(r) = V_{\nu_i\nu_i}(r),$$

Some approximations later ....

$$V_{PNC}^{\text{loop}} \approx \sum_i \frac{G_A}{m_e}\left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)\left[(2\vec{\sigma}_p \cdot \vec{p}_e)V_{\nu_i\nu_i}(r) + (\vec{\sigma}_e \times \vec{\sigma}_p)\cdot\vec{\nabla}V_{\nu_i\nu_i}(r)\right]$$

THE PHOTON PENGUIN

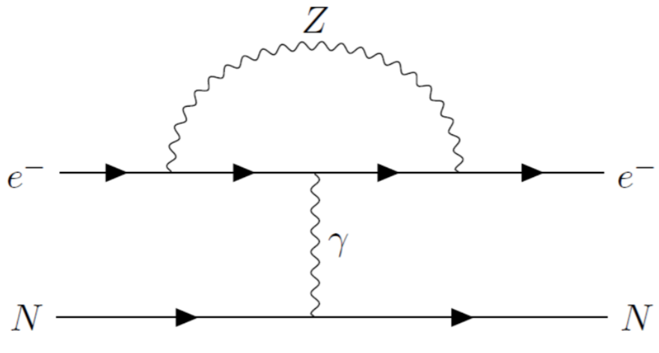
$$H_1 = H_1^{\text{penguin}} = (g_V^e)(g_A^e)G_F\, \alpha\, m_e,$$

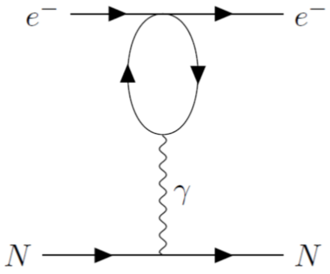
$$H_2 = H_2^{\text{penguin}} = 0,$$

$$C = C^{\text{penguin}} = 0,$$

$$F(r) = F^{\text{penguin}}(r)$$

The penguin is **short ranged**, since effectively, it's like two-electron exchange, or like two-proton exchange if you connect the Z boson to the proton legs.





$$F^{\text{penguin}}(r) \sim 12\Gamma\left(\frac{3}{2}\right)\sqrt{m_e}\,\frac{e^{-2m_e r}}{r^{5/2}}$$

Technical

EFFECT OF THE FORCE: TRUE EIGENSTATES

When Parity is conserved

$$|n, l, m\rangle \otimes \begin{cases} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \\ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \end{cases}$$

Eigenstates of L^2, L_z

Eigenstates of total spin

When parity is violated

Eigenstates are now states of definite total angular momentum:

$$\vec{F} \equiv \vec{L}_e + \vec{S}_e + \vec{S}_p$$

$$|n, f, m_f, j, \ell\rangle$$

Technical

WHAT WE WANT TO COMPUTE

$$R = \mathcal{I}m\left(\frac{E1_{PV}}{M1}\right)$$

So, pick some states and we're good to go?

$$\frac{\langle A' | \text{Electric Dipole} | B' \rangle}{\langle A' | \text{Magnetic Dipole} | B' \rangle} \approx \frac{\langle A' | \text{Electric Dipole} | B' \rangle}{\langle A | \text{Magnetic Dipole} | B \rangle}$$

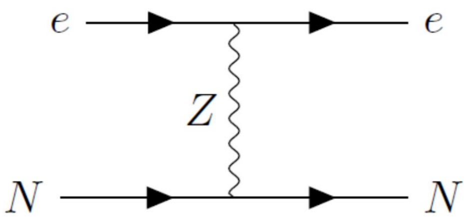
Not quite yet..

A FEW ISSUES

The tree: exists

Neutrino loop: uh oh!

The tree level diagram cannot be ignored!



And in fact, it dominates over the loop for low ℓ states

The neutrino potential is highly singular

We used the 4-Fermi approximation (a non-renormalizable interaction), so we expect it to not work at high energies or low ℓ.

Can work out neutrino force in the full theory but Z diagram dominates anyway.

Instead, just find some values of ℓ for which our approximation works..

TWO BIRDS WITH ONE STONE

ℓ	From V_{PNC}^{tree}	V_{PNC}^{loop}
$\ell = 0$	$\sim \alpha^5 \left(\frac{m_e}{m_Z}\right)^2$	does not converge
$\ell = 1$	$\sim \alpha^7 \left(\frac{m_e}{m_Z}\right)^4$	does not converge
$\ell \geq 2$	$\sim \alpha^{2l+5} \left(\frac{m_e}{m_Z}\right)^{2\ell+2}$	$\sim \alpha^8 \left(\frac{m_e}{m_Z}\right)^4$

For $\ell = 2$,

$$\frac{\mathcal{M}_{\text{tree}}}{\mathcal{M}_{\text{loop}}} \sim \alpha \left(\frac{m_e}{m_Z}\right)^2 \approx 10^{-13}.$$

Table of matrix elements $\langle n, \ell + 1, m|V|n', \ell, m'\rangle$ from the tree potential and loop potential

THE SIMPLEST POSSIBLE RESONANCE

Technical

$|n, f, m_f, j, \ell\rangle$

At $\ell = 3$

$$|A\rangle = |4,3,3,5/2,3\rangle \equiv -\frac{1}{\sqrt{7}}\psi_{432}|\uparrow\uparrow\rangle + \sqrt{\frac{6}{7}}\psi_{433}|\downarrow\uparrow\rangle,$$

$$|B\rangle = |4,3,3,7/2,3\rangle \equiv -\frac{1}{2}\sqrt{\frac{3}{7}}\psi_{432}|\uparrow\uparrow\rangle + \frac{1}{2}\sqrt{\frac{7}{2}}\psi_{433}|\uparrow\downarrow\rangle - \frac{1}{2\sqrt{14}}\psi_{433}|\downarrow\uparrow\rangle,$$

$$|\Delta\rangle = |4,3,3,5/2,2\rangle \equiv \psi_{422}|\uparrow\uparrow\rangle,$$

$$\psi_{nlm} = \langle r,\theta,\phi|nlm\rangle = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} e^{-r/na_0} \left[L^{2l+1}_{n-l-1}(2r/na_0)\right] Y_l^m(\theta,\phi).$$

GOOD OLD PERTURBATION THEORY

True Eigenstates:

$$|A'\rangle = |A\rangle + \frac{\langle\Delta|V_{PNC}|A\rangle}{E_A - E_\Delta}|\Delta\rangle + \cdots$$

$$|B'\rangle = |B\rangle + \frac{\langle\Delta|V_{PNC}|B\rangle}{E_B - E_\Delta}|\Delta\rangle + \cdots$$

Need to be careful and consider fine structure corrections, hyperfine splitting etc, since this effect is smaller than all these QED effects

$$\frac{\langle A'|\text{Electric Dipole}|B'\rangle}{\langle A'|\text{Magnetic Dipole}|B'\rangle} \approx \frac{\langle A'|\text{Electric Dipole}|B'\rangle}{\langle A|\text{Magnetic Dipole}|B\rangle}$$

Big Numbers!

$$R = \frac{-7\alpha m_e^3 m_p G_A G_F^2 \left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)}{302778777600\pi^3 g_p (29 g_p m_e - 21609000 m_p)}$$
$$\times \left[(24335 g_p m_e - 17503290000 m_p) + \nu_i^2 (3858 g_p m_e + 84015792000 m_p)\right] + \mathcal{O}(\nu_i^4),$$

NEUTRINO INDUCED OPTICAL ROTATION FOR STATES $\ell \geq 2$

$$\nu_i \equiv \frac{1}{\alpha}\frac{m_{\nu_i}}{m_e}$$

This is: a_0/λ_{ν_i}

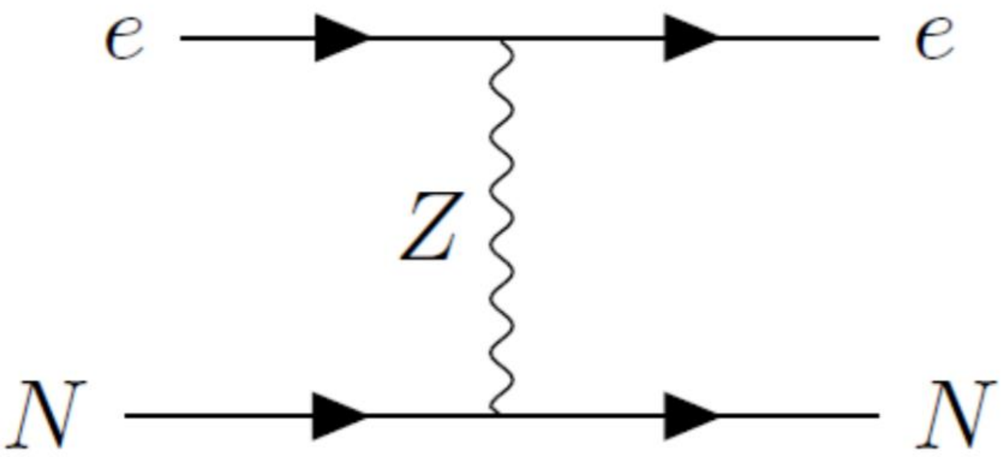
a_0 = Bohr Radius of hydrogen

λ_{ν_i} = Compton wavelength of neutrino mass eigenstate i

$$R = \mathcal{I}m\left(\frac{E1_{PV}}{M1}\right) \approx \left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)\left(-7.7\times 10^{-33} + 3.7\times 10^{-32}\nu_i^2\right)$$

But Sadly in the denominator !!

FOR PERSPECTIVE



When the tree diagram dominates:

$$R = \mathcal{I}m\left(\frac{E1_{PV}}{M1}\right) \sim 10^{-10}$$

Remember: These are for lower ℓ states. For high ℓ states, the loop dominates.

TO CONCLUDE...

Downsides:

Seems too small to measure at present.

Applicable to states of higher angular momentum in hydrogen that may be Boltzmann suppressed.

Upsides:

The neutrino force has the longest range of all parity violating forces in atomic systems.

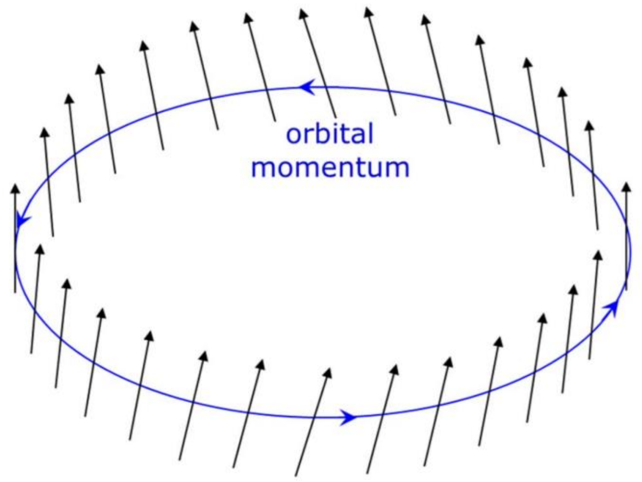
In hydrogen the effect is small, but it can be larger in other systems, for instance, larger atomic number **enhances** the effect.

SO HOW DO NEUTRINOS MAKE HYDROGEN SWEET?

The parity violating force mixes up radial and spin degrees of freedom.

Position dependent spin configuration

$$H_{HPV} \propto \vec{\sigma}_N \cdot \vec{p}_N \Rightarrow$$ spin tilted along momentum



Now both hydrogen and sugar are helical!

NEUTRINO FORCES IN NEUTRINO BACKGROUNDS

Can we do better than $1/r^5$?

MODIFIED FERMION PROPAGATOR IN A BACKGROUND

In vacuum

$$\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$$

In a background

I NEED SOME SPACE!

MODIFIED FERMION PROPAGATOR IN A BACKGROUND

$$(\not{p}+m)\left\{\frac{i}{p^2-m^2+i\epsilon}-(2\pi)\delta\left(p^2-m^2\right)\left[\Theta\left(p^0\right)n_+(\mathbf{p})+\Theta\left(-p^0\right)n_-(\mathbf{p})\right]\right\}$$

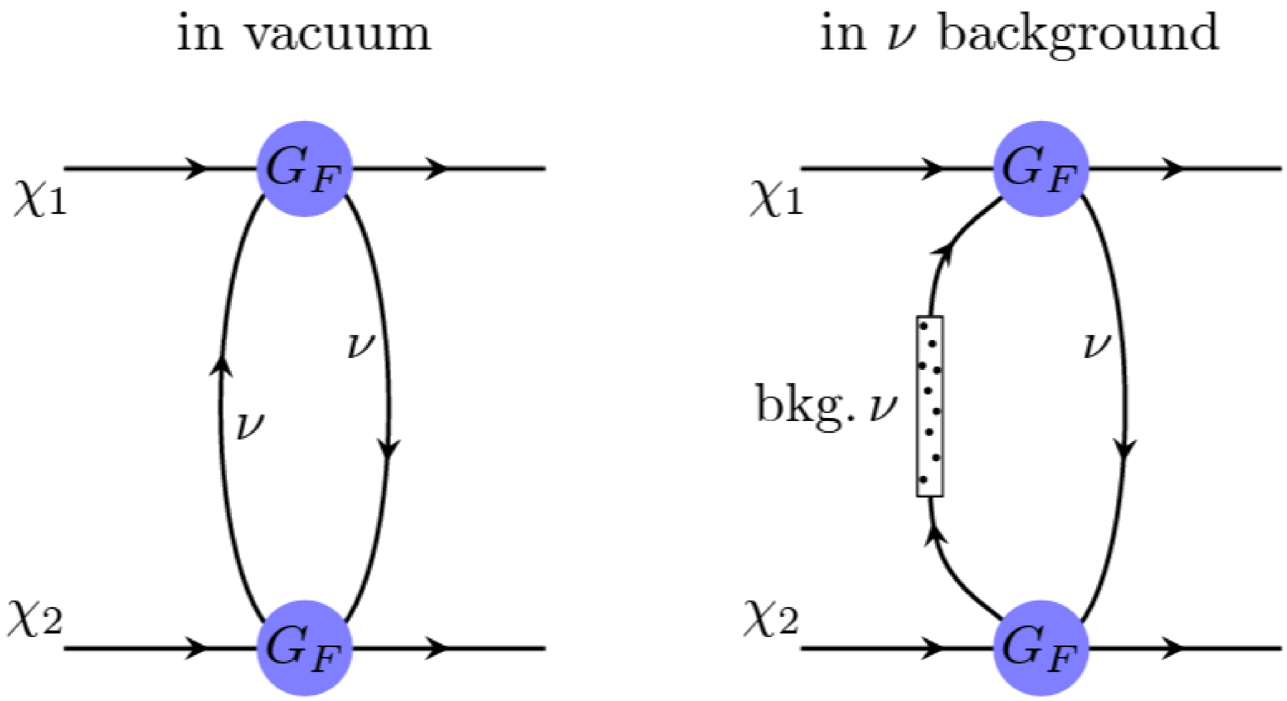
Vacuum propagator

Background correction

- In the background potential, the delta function puts the fermion on shell!

- Proportional to the density of fermions and anti-fermions in the background.

THE MODIFIED NEUTRINO FORCE

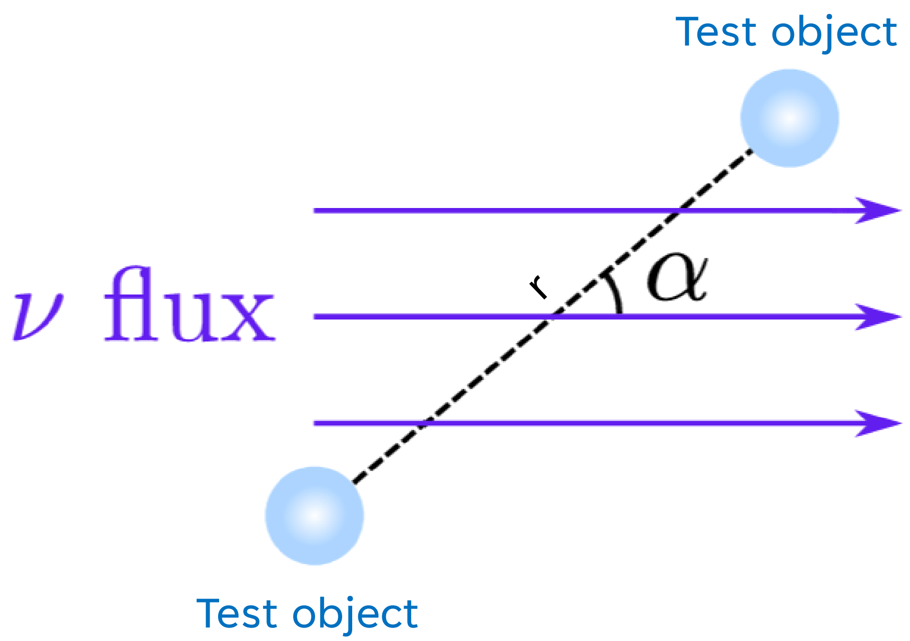


DIRAC VS. MAJORANA: A DREAM

To distinguish between Dirac and Majorana by measuring force between them.

	$r \ll T^{-1}$	$r \gg T^{-1}$
Dirac:	$-\frac{14.4}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{m_\nu T^3}{r}$	$-\frac{1}{32\pi^3} G_F^2 g_V^1 g_V^2 \frac{m_\nu}{T} \frac{1}{r^5}$
Majorana:	$-\frac{248.9}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{T^5}{m_\nu r}$	$-\frac{1}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{1}{m_\nu T} \frac{1}{r^7}$

MONOCHROMATIC DIRECTIONAL BACKGROUNDS

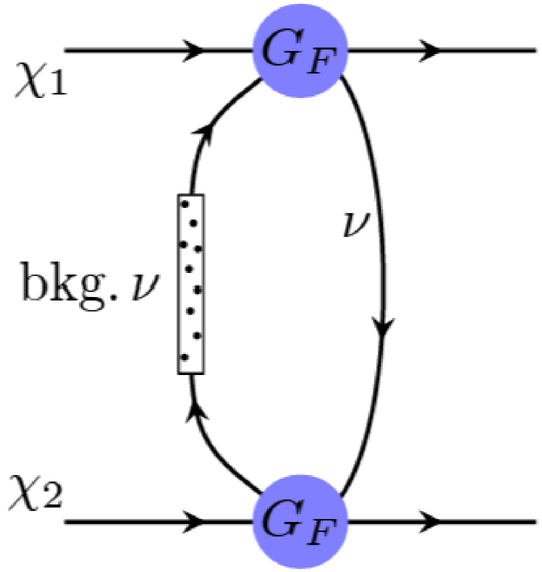


Consider a beam with the following properties:

1. Monochromatic beam with all neutrinos travelling in the same direction
2. Beam energy $E_\nu \gg m_\nu$, neutrino mass being ignored in this talk.
3. Beam flux density ϕ_0

Guess Before You Calculate!

NAÏVE EXPECTATIONS



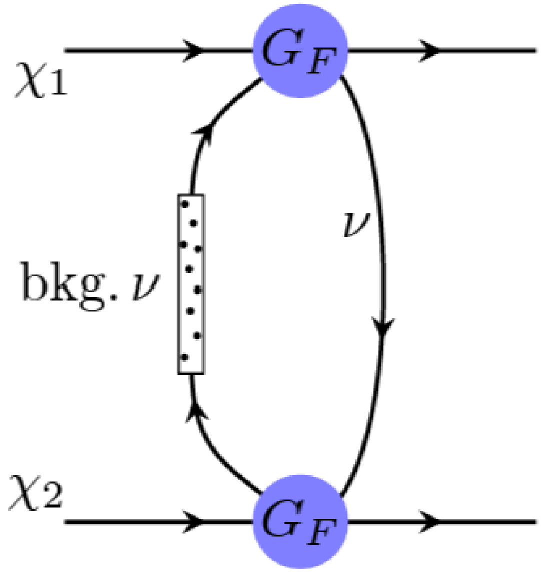
Oscillations

Exchange of a “real” neutrino, as opposed to a virtual one gives oscillations

Radial dependence

From “geometry” of one virtual particle exchange, expect $1/r$

NAÏVE EXPECTATIONS



Oscillations

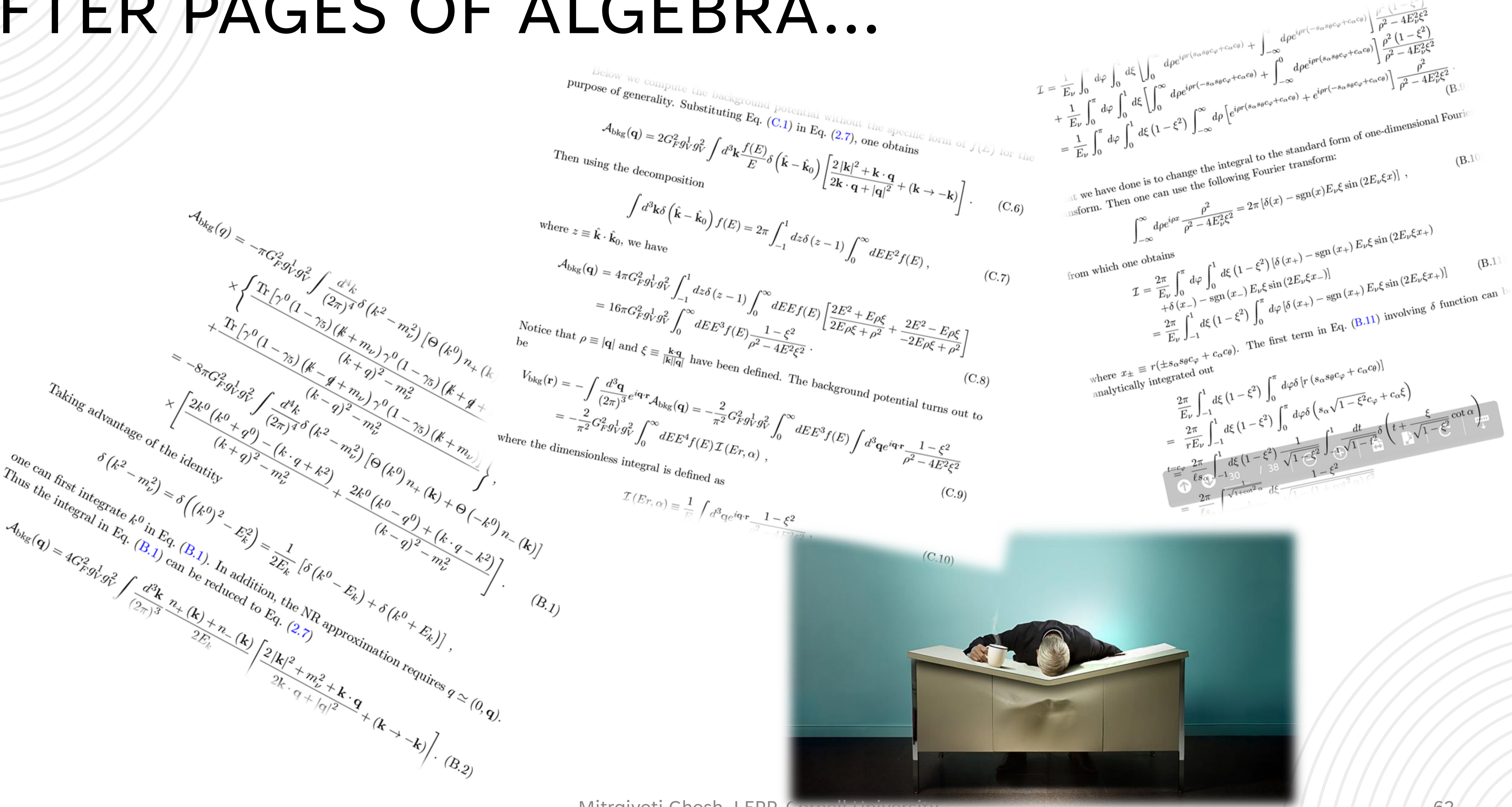
Exchange of a “real” neutrino, as opposed to a virtual one gives oscillations

Radial dependence

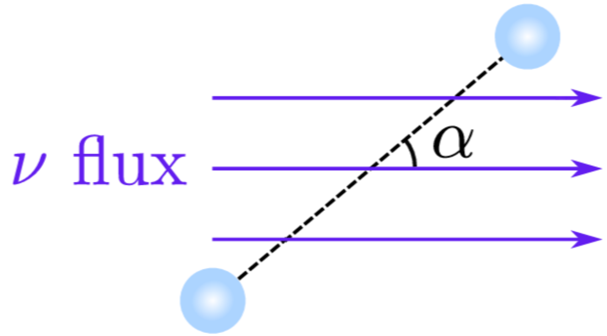
From “geometry” of one virtual particle exchange, expect $1/r$

$$V(r) \sim \frac{e^{-2iE_\nu r f(\alpha)}}{r} \sim \frac{1}{r}\cos(2E_\nu r f(\alpha))$$

AFTER PAGES OF ALGEBRA...



THE LEADING LONG-DISTANCE BEHAVIOR



Vectorial Couplings

Beam parameters

Radial dependence

Oscillations

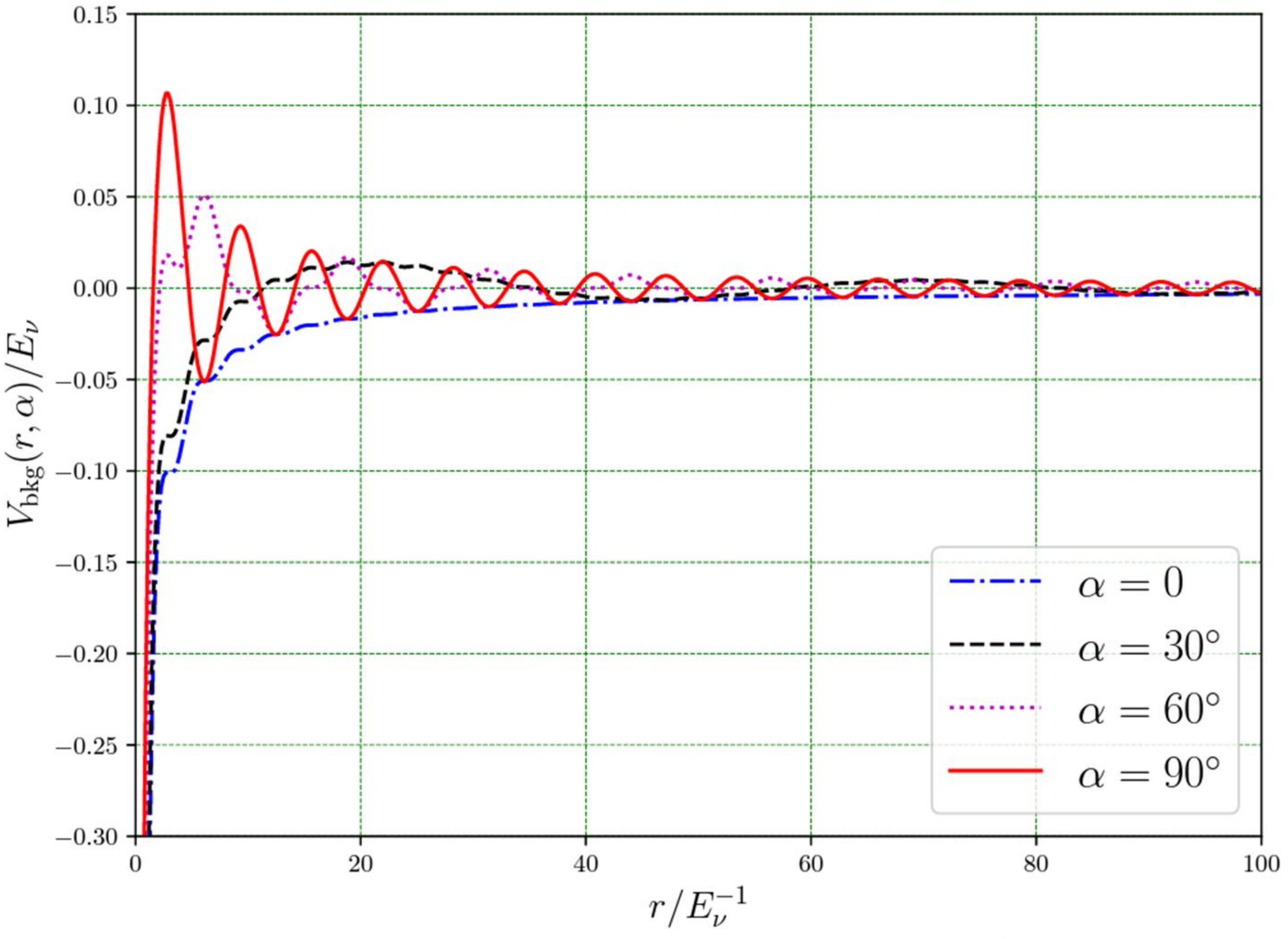
$$V_{\rm bkg}\left(r \gg E_\nu^{-1}, \alpha \ll 1\right) = -\frac{g_V^1 g_V^2}{\pi} G_F^2 \times \Phi_0 E_\nu \times \frac{1}{r} \times \cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$

Non-leading terms that go as $1/r^2$ etc. exist, but are negligible at large distances

INTUITION WAS:

$$V(r) \sim \frac{e^{-2iE_\nu r f(\alpha)}}{r} \sim \frac{1}{r}\cos(2E_\nu r f(\alpha))$$

THE GENERAL FORM OF $V_{bkg}(r)$



Expression just a bunch of math!

But....

See next slide for nice limits!

FORCE BETWEEN ATOMS AT $\alpha = 0$

Number density of electron neutrinos in the beam

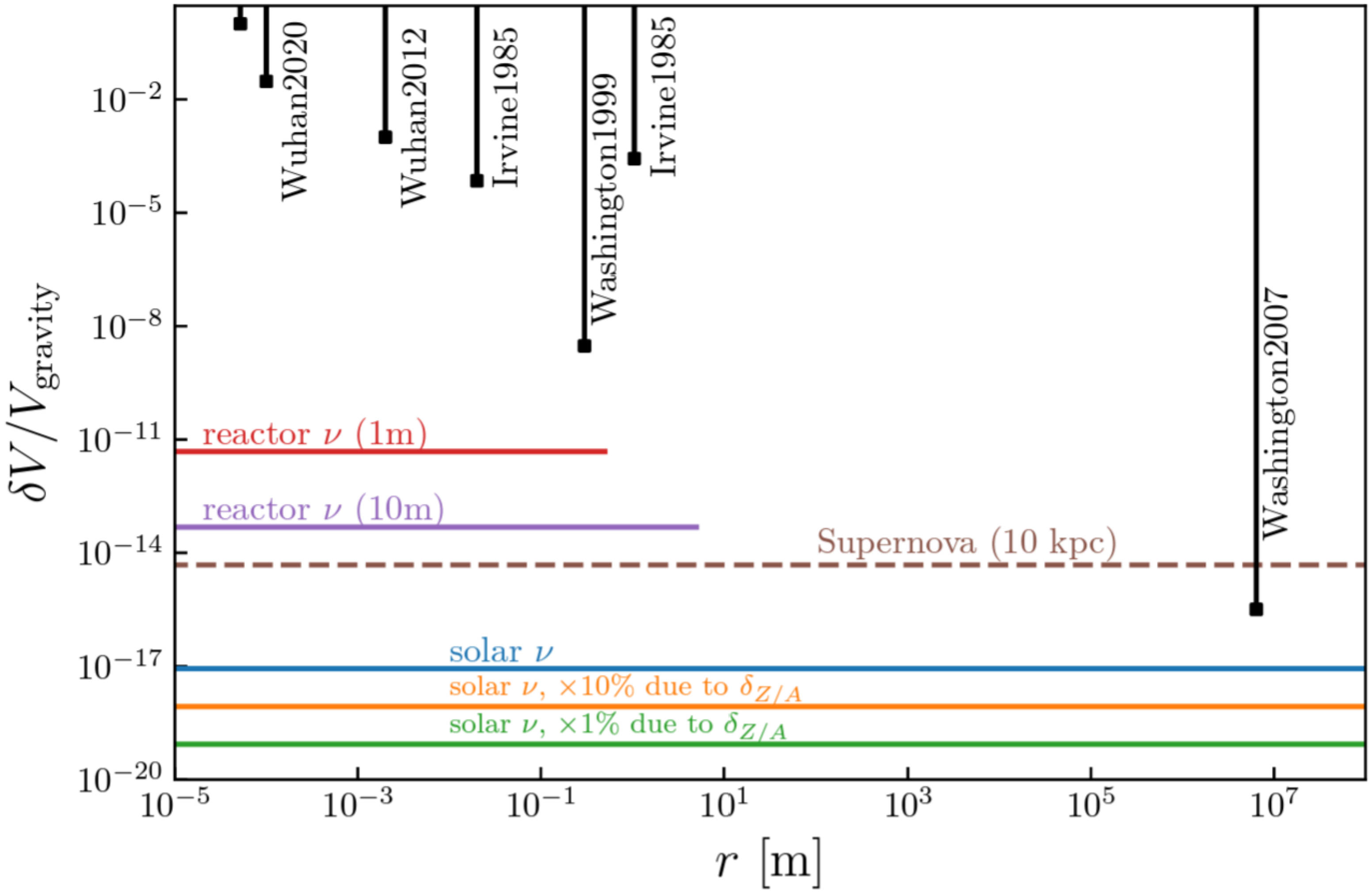
$$V_{\rm bkg}(r) = -\frac{G_F^2 \Phi E_\nu}{\pi r}\frac{m_1 m_2}{m_p^2} \times f(A_1, A_2, Z_1, Z_2, n_e)$$

$$f(A_1, A_2, Z_1, Z_2, n_e) = \frac{1}{4}\left[n_e\left(\frac{3Z_1}{A_1} - 1\right)\left(\frac{3Z_2}{A_2} - 1\right) + (1 - n_e)\left(1 - \frac{Z_1}{A_1}\right)\left(1 - \frac{Z_2}{A_2}\right)\right]$$

$$\frac{V_{\rm bkg}(r)}{V_{\rm grav}(r)} \sim 10^{-13}$$

Can fifth-force Experiments achieve this sensitivity in the future?

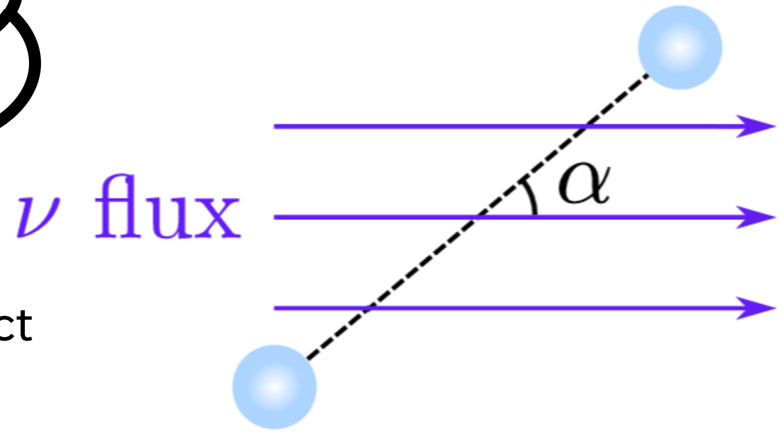
Currently have a sensitivity of 10^{-9}, just 4 orders of magnitude above.



Wuhan2020
Wuhan2012
Irvine1985
Washington1999
Irvine1985
Washington2007
reactor ν (1m)
reactor ν (10m)
Supernova (10 kpc)
solar ν
solar ν, $\times 10\%$ due to $\delta_{Z/A}$
solar ν, $\times 1\%$ due to $\delta_{Z/A}$
$\delta V / V_{\rm gravity}$
r [m]
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-20}
10^{-5}
10^{-3}
10^{-1}
10^{1}
10^{3}
10^{5}
10^{7}

HOWEVER..

$$\cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$



- Considering macroscopic objects: the **finite size** of the object means that the net force between them will require an integration over the angle α, which can kill the leading $1/r$ dependence.

$$\Delta(\alpha^2) \lesssim (E_\nu r)^{-1}$$

- In realistic scenarios there will be a certain energy distribution in the neutrino beam, which can also smear out the force and kill the $1/r$ dependence.

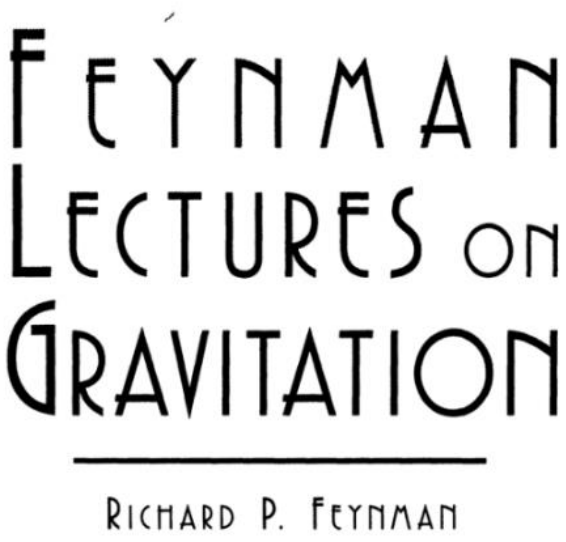
Assuming that the beam is truly monochromatic, we find, for test objects of size R:

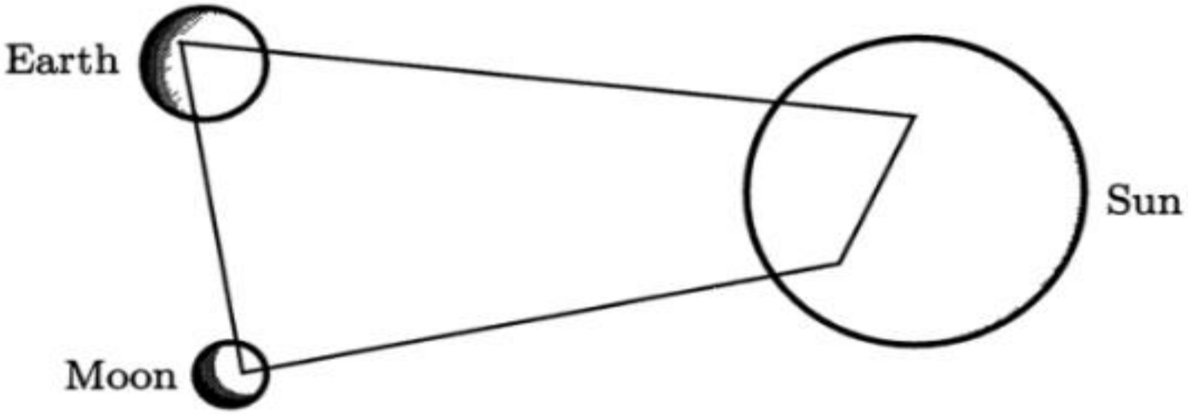
$$\alpha \lesssim (E_\nu R)^{-1}$$

If hydrogen atoms are used as test objects, we need $\alpha \ll 10^{-2}$.

NEED LOWER ENERGY BACKGROUNDS!

The neutrino-force as an explanation for gravity??





$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \quad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi \rho(R) R^2 dR}{2R^3}, \quad (2.4.5)$$

Background!

US: $r^{-5} \rightarrow 1/r$ **FEYNMAN:** $r^{-3} \rightarrow 1/r$

CONCLUSIONS (PART 2)

1. Background effects can **greatly enhance static forces** obtained from particle exchange.

2. In the context of the two-neutrino force, the presence of a directional background takes the radial dependence **from a $1/r^5$ to a $1/r$**!

3. The $1/r$ dependence, however, is fragile and is easily killed by the smearing effects of finite size of the objects and/or the energy spread in the background neutrino beam.

4. Still, seems exciting as a way to finally probe the neutrino-force that has eluded us so far!

WHAT NEXT?

1. Computation of the force outside 4-Fermi regime.
2. Incorporate neutrino mass and mixing.
3. Separate calculations for Dirac and Majorana neutrinos? Maybe possible to distinguish them?
4. Most importantly, try to design some experiments that can one day hope to probe the **longest range 2-fermion mediated force** in the Standard Model.

Thanks!!